

# An Overview on Stability Methods, with Applications

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**Abstract.** This mini-course presents an overview of stochastic stability methods, mostly motivated by (but not limited to) stochastic network applications. We work with stochastic recursive sequences, and, in particular, Markov chains, in a general Polish state space. We discuss and compare methods based on

- (i) Lyapunov functions,
- (ii) fluid limits,
- (iii) explicit coupling (renovating events and Harris chains),
- (iv) monotonicity,

and some others. We also discuss existence of stationary solutions and instability methods.

Lectures are based on handouts of my lecture notes (Colorado State Uni, 96; Novosibirsk State Uni, 97-99), on our joint overview paper with Takis Konstantopoulos (2004), and on some recent publications.