273018 Special Functions, Homework #11, 29.3. 2011 TUESDAY

This is the last set of homeworks

Final Exam: Wednesday April 20, at 8.45–13

Last Exercise session: Wednesday 30.3 Hilbertrummet

42 **13.2.3** From the generating function derive the Rodrigues representation

$$L_n^k(x) = \frac{e^x x^{-k}}{n!}\frac{d^n}{dx^n}(e^{-x}x^{n+k}).$$

43 **13.2.6** Expand e^{-ax} in a series of associated Laguerre polynomials $L_n^k(x)$, k fixed and n ranging from 0 to ∞.

(a) Evaluate directly the coefficients in your assumed expansion.

(b) Develop the desired expansion from the generating function.

$$ANS.\quad e^{-ax} = \frac{1}{(1+a)^{1+k}}\sum_{n=0}^{\infty}\left(\frac{a}{1+a}\right)^n L_n^k(x), \qquad 0 \le x < \infty.$$

44 **13.3.6** $V_n(x) = (1-x^2)^{1/2}U_{n-1}(x)$ is not defined for $n = 0$. Show that a second and independent solution of the Chebyshev differential equation for $T_n(x)$, $(n = 0)$ is $V_0(x) = \arccos x$ (or $\arcsin x$).

45 **13.3.27** Develop the following Chebyshev expansions (for $[-1, 1]$):

$$\text{(b)}\quad \left.\begin{array}{ll} +1, & 0 < x \le 1 \\ -1, & -1 \le x < 0 \end{array}\right\} = \frac{4}{\pi}\sum_{s=0}^{\infty}(-1)^s(2s+1)^{-1}T_{2s+1}(x).$$

Also draw a picture and compare the result to homework 29.

Final Exam: The table "Matemaattisia Erikoisfunktioita" will be allowed in the exam, but no other literature.

Good Luck in the Exam!

Lähteet: Kuten aikaisemmin

11. TŠEBYSEVIN POLYNOMIT:

11.1. $(1-x^2)y''(x) - xy'(x) + n^2y(x) = 0$

11.2. $T_n(x) = \cos(n\arccos x)$

11.3. $U_n(x) = (1-x^2)^{-\frac{1}{2}}\sin((n+1)\arccos x) = (1-x^2)^{-\frac{1}{2}}V_{n+1}(x)$

11.4. $2T_n(x)\,T_m(x) = T_{n+m}(x) + T_{n-m}(x)$, $n \ge m$

11.5. $$\int_{-1}^{1} T_m(x)T_n(x)\frac{dx}{\sqrt{1-x^2}} = \begin{cases} 0 & , m \ne n \\ \frac{\pi}{2} & , m = n \ne 0 \\ \pi & , m = n = 0 \end{cases}$$

11.6. $T_0(x) = 1$

$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

$T_3(x) = 4x^3 - 3x$

$T_4(x) = 8x^4 - 8x^2 + 1$

$T_5(x) = 16x^5 - 20x^3 + 5x$

$$T_n(x) = \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{n!\,x^{n-2k}(x^2-1)^k}{(2k)!\,(n-2k)!}$$

11.7. $T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0$

11.8. $$x^{2n} = \frac{1}{2^{2n-1}}\cdot\sum_{k=0}^{n-1}\binom{2n}{k}T_{2n-2k}(x) + \frac{1}{2^{2n-2}}\binom{2n}{n}T_0(x)$$

$$x^{2n+1} = \frac{1}{2^{2n}}\sum_{k=0}^{n}\binom{2n+1}{k}T_{2n+1-2k}(x)$$

Lähteet: Kuten aikaisemmin