Ex 1 – Löse $Au = b_1$, $Av = b_2$, $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Metod 1:

$$\left(\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 1 \\ 2 & 3 & -1 & 0 & 1 \end{array}\right) \xrightarrow{B01^+} \left(\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 1 \\ 0 & -1 & -3 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{B03} \left(\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & 1 \end{array}\right) \xrightarrow{B01^-} \left(\begin{array}{ccc|cc} 1 & 0 & -5 & -3 & -1 \\ 0 & 1 & 3 & 2 & 1 \end{array}\right)$$

u_3, v_3 fria variabler, Sätt $u_3 = s$, $v_3 = t$

$$\begin{cases} u_1 = -3 + 5s \\ u_2 = 2 - 3s \\ u_3 = s, \quad s \in \mathbb{R} \end{cases}, \qquad \begin{cases} v_1 = -1 + 5t \\ v_2 = 1 - 3t \\ v_3 = t, \quad t \in \mathbb{R} \end{cases}.$$

Metod 2:

1°) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 3 & -1 & 0 \end{array}\right) \xrightarrow{B01^+} \begin{matrix} \\ (2) \end{matrix}\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & -3 & -2 \end{array}\right)^{(E)}$ $\therefore$ $A = LU = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \end{pmatrix}$

$\xrightarrow{B03} \dots \xrightarrow{B01^-} \left(\begin{array}{ccc|c} 1 & 0 & -5 & -3 \\ 0 & 1 & 3 & 2 \end{array}\right)$ $\therefore$ $\begin{cases} u_1 = -3 + 5s \\ u_2 = 2 - 3s \\ u_3 = s \end{cases}, \ s \in \mathbb{R}$

2°) a) Löser $Ly = b_2$.

$$(L \mid b_2) = \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 2 & 1 & 1 \end{array}\right) \xrightarrow{B01^+} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right) \quad \therefore \ y = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

b) Löser $Uv = y$.

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & -3 & -1 \end{array}\right) \xrightarrow[B01^-]{B03} \begin{array}{c} \quad \quad F \\ \left(\begin{array}{ccc|c} 1 & 0 & -5 & -1 \\ 0 & 1 & 3 & 1 \end{array}\right) \end{array}$$

$$\therefore \begin{cases} v_1 = -1 + 5t \\ v_2 = 1 - 3t \\ v_3 = t, \quad t \in \mathbb{R} \end{cases}.$$

Ex 18a)/3 Bestäm den symmetrisk LDU-faktoriseringen $A = LDL^T$ av

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 6 & 4 \\ 0 & 4 & 11 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 6 & 4 \\ 0 & 4 & 11 \end{pmatrix} \xrightarrow{Bo1^+} \begin{matrix} (2) \\ (0) \\ \end{matrix}\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 11 \end{pmatrix} \xrightarrow{Bo2^+} \begin{matrix} \\ \\ (2) \end{matrix}\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}}_{U}, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\underline{A = LDU = LDL^T}.$$

Ex. 4.3. $E = \{ f : \mathbb{R} \to \mathbb{R}^n,\ D_f = [a,b] \}$.

$$\left.\begin{array}{l} f, g \in E \\ x \in [a,b] \end{array}\right\} \Rightarrow \quad \begin{array}{l} f(x) = (f_1(x), \dots, f_n(x)), \\ g(x) = (g_1(x), \dots, g_n(x)), \\ f_1, \dots, f_n, g_1, \dots, g_n : \mathbb{R} \to \mathbb{R}, \\ \text{Def. mängd } [a,b] \end{array}$$

$$\forall x \in [a,b] \begin{cases} \underset{\in E}{(f+g)}(x) := (f_1(x)+g_1(x), \dots, f_n(x)+g_n(x)) \quad (\in \mathbb{R}^n) \\ \underset{\in E}{(\lambda f)}(x) := (\lambda \cdot f_1(x), \dots, \lambda f_n(x)) \\ \underset{\in E}{0}(x) := (0, 0, \dots, 0) (\in \mathbb{R}^n) \end{cases}$$

Axiomen lätta att verifiera.

Ex. 1. $0 \in E$ entydig.

$0_1, 0_2 \in E$ nollvektorer $\Rightarrow$
$$\underline{0_1} = 0_1 + 0_2 \quad (0_2 \text{ nollv.})$$
$$= 0_2 + 0_1 \quad (\text{I (a)})$$
$$\underline{=} \ 0_2. \quad (0_1 \text{ nollv.})$$

$\therefore \ \underline{0_1 = 0_2}$

2. Givet $x \in E$ är $-x$ entydig.

Ant. $x + (-x) = 0$ och $x + (-y) = 0$, $-x, -y \in E$.

$$\underline{-x =} -x + 0 = -x + (x + (-y)) = (-x + x) + (-y)$$
$$= (x + (-x)) + (-y) = 0 + (-y) = -y + 0 \ \underline{= -y}.$$

$\therefore \ \underline{-x = -y}.$

Ex 4.4. $U = \{(x_1 \; 0 \; x_3) : x_1, x_3 \in \mathbb{R}\} \subseteq \mathbb{R}^3$.

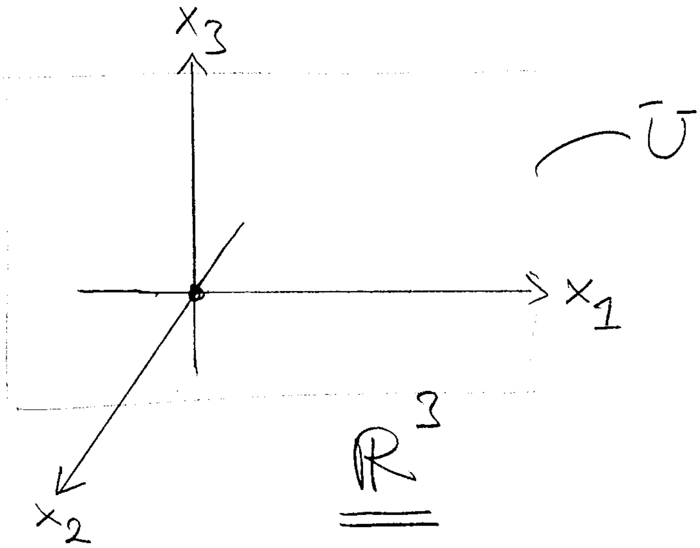
1°) $(a \; 0 \; b), (c \; 0 \; d) \in U$

$\Rightarrow (a \; 0 \; b) + (c \; 0 \; d) = (a+c \; 0 \; b+d) \in U$

2°) $(a \; 0 \; b) \in U$, $\lambda \in \mathbb{R}$

$\Rightarrow \lambda (a \; 0 \; b) = (\lambda a \; 0 \; \lambda b) \in U$

$\therefore$ 1°) och 2°) $\Rightarrow$ <u>U underrum av $\mathbb{R}^3$</u>.



<u>$0 = (0,0,0)$ i $\mathbb{R}^3$</u>
<u>tillhör U</u>.

<u>U "=" $x_1 x_3$-planet</u>.