Ex

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$ 2/3 - matris.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$$ 3/2 - matris.

$$O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ Nollmatris av typ 3/3.

$$a = \begin{pmatrix} 2 & 3 & 4 & 5 \end{pmatrix}$$ radvektor, matris av typ 1/4.

$$b = \begin{pmatrix} 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{pmatrix}$$ kolonnvektor, matris av typ 5/1.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$ Diagonalmatris av typ 3/3.

$$I = I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ Enhetsmatris av typ 4/4

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 5 \end{pmatrix}$$ A uppåt triangulär, B nedåt triangulär

Ex $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 2 & 5 \end{pmatrix}$

$$\underline{A + B} = \begin{pmatrix} 1+2 & 2+3 & 3+0 \\ 0+(-1) & 1+2 & 4+5 \end{pmatrix} = \underline{\begin{pmatrix} 3 & 5 & 3 \\ -1 & 3 & 9 \end{pmatrix}}.$$

$$\underline{A - B} = A + (-1)\cdot B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix} + \begin{pmatrix} -2 & -3 & 0 \\ 1 & -2 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 1+(-2) & 2+(-3) & 3+0 \\ 0+1 & 1+(-2) & 4+(-5) \end{pmatrix}$$

$$= \underline{\begin{pmatrix} -1 & -1 & 3 \\ 1 & -1 & -1 \end{pmatrix}}.$$

Ex Associationslagen. $A = (a_{ik})$, $B = (b_{ik})$, $C = (c_{ik})$ alla av typ m/n.

$$\underline{(A+B)+C} = (a_{ik}+b_{ik}) + (c_{ik})$$
$$= ((a_{ik}+b_{ik}) + c_{ik})$$
$$= (a_{ik} + (b_{ik}+c_{ik}))$$
$$= (a_{ik}) + (b_{ik}+c_{ik})$$
$$\underline{= A + (B+C)}$$

Ex $A + B = A \implies (-1)\cdot A + (A+B) = (-1)\cdot A + A$
$\implies (A + (-1)\cdot A) + B = A + (-1)\cdot A$
$\implies B = 0$

$(A + (-1)A = 0$ och $A + B = 0)$
$\implies A + (-1)A = A + B$
$\implies (-1)A + (A + (-1)A) = (-1)A + (A+B)$
$\implies (-1)A = B$

Ex $\underline{\alpha(A+B)} = \alpha(a_{ik}+b_{ik}) = (\alpha(a_{ik}+b_{ik}))$
$= (\alpha a_{ik} + \alpha b_{ik}) = (\alpha a_{ik}) + (\alpha b_{ik})$
$\underline{= \alpha A + \alpha B}.$

$$(\alpha+\beta)A = ((\alpha+\beta)a_{ik}) = (\alpha a_{ik} + \beta a_{ik})$$
$$= (\alpha a_{ik}) + (\beta a_{ik}) = \alpha \cdot A + \beta \cdot B$$

Ex

$$\begin{cases} x_1 - 2x_2 + x_3 = 2 \\ 2x_1 + x_2 - 2x_3 = 3 \\ 3x_1 - x_2 + 4x_3 = 4 \end{cases} \quad (1)$$

<u>Sätt</u>: $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 3 & -1 & 4 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

Vill uttrycka (1) som $Ax = b$.

<u>Alltså</u>:

$$Ax = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 3 & -1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 + x_3 \\ 2x_1 + x_2 - 2x_3 \\ 3x_1 - x_2 + 4x_3 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 \\ 2x_1 \\ 3x_1 \end{pmatrix} + \begin{pmatrix} -2x_2 \\ x_2 \\ -1x_2 \end{pmatrix} + \begin{pmatrix} x_3 \\ -2x_3 \\ 4x_3 \end{pmatrix}$$

$$= x_1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} + x_3\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

Ax <u>linjärkombination</u> av kolonnerna i A.

Ex

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad \underline{2/3\text{-matris}}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad 3/2\text{-matris}$$

$$\underline{AB} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1\cdot 1 + 2\cdot 0 + 3\cdot 1 & 1\cdot 0 + 2\cdot 1 + 3\cdot 1 \\ 4\cdot 1 + 5\cdot 0 + 6\cdot 1 & 4\cdot 0 + 5\cdot 1 + 6\cdot 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 10 & 11 \end{pmatrix}$$

$$\underline{2/2\text{-matris}}$$

lika

$$(2/3)\cdot(3/2)$$

<u>resultat</u>: 2/2-matris

$$\underline{BA} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 1\cdot 1 + 0\cdot 4 & 1\cdot 2 + 0\cdot 5 & 1\cdot 3 + 0\cdot 6 \\ 0\cdot 1 + 1\cdot 4 & 0\cdot 2 + 1\cdot 5 & 0\cdot 3 + 1\cdot 6 \\ 1\cdot 1 + 1\cdot 4 & 1\cdot 2 + 1\cdot 5 & 1\cdot 3 + 1\cdot 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \qquad \underline{3/3\text{-matris}}$$

lika

$$(3/2)\cdot(2/3)$$

<u>resultat</u>: 3/3-matris

$\therefore$ $\underline{AB \neq BA}$, vilket <u>vanligen är fallet</u>.

Ex A av typ m/n, $A = (a_1 \; \dots \; a_n)$

$$A = \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_m \end{pmatrix}$$

1°) $\underline{I \cdot A} = I_m \cdot A = (I_m a_1 \; \dots \; I_m a_n) = (a_1 \; \dots \; a_n) \underline{= A}.$

2°) $\underline{A \cdot I} = A \cdot I_n = \begin{pmatrix} \bar{a}_1 I_n \\ \vdots \\ \bar{a}_m I_n \end{pmatrix} = \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_m \end{pmatrix} = A.$

Ex $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 6 \\ -1 & -3 \end{pmatrix}$

$$A \cdot B = \begin{pmatrix} 1\cdot 2 + 2\cdot(-1) & 1\cdot 6 + 2\cdot(-3) \\ 3\cdot 2 + 6\cdot(-1) & 3\cdot 6 + 6\cdot(-3) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 2\cdot 1 + 6\cdot 3 & 2\cdot 2 + 6\cdot 6 \\ -1\cdot 1 - 3\cdot 3 & -1\cdot 2 + (-3)\cdot 6 \end{pmatrix} = \begin{pmatrix} 20 & 40 \\ -10 & -20 \end{pmatrix}$$

$$\underline{A \cdot B \neq B A}$$

och

$$\underline{A \cdot B = 0 \not\Rightarrow (A = 0 \vee B = 0)}$$

$$\underline{A \cdot B = A \cdot C \not\Rightarrow B = C} \quad \begin{pmatrix} \text{välj t. ex.} \\ C = 0 \end{pmatrix}$$

Ex

(i) $((A+B)^T)_{ki} = (A+B)_{ik} = A_{ik} + B_{ik}$

$= (A^T)_{ki} + (B^T)_{ki}$

$= (A^T + B^T)_{ki}$

$\therefore \underline{(A+B)^T = A^T + B^T}.$

(ii) $((\lambda A)^T)_{ki} = (\lambda A)_{ik} = \lambda \cdot A_{ik} = \lambda \cdot (A^T)_{ki}$

$\therefore \underline{(\lambda A)^T = \lambda \cdot A^T}.$

Ex   Sats 2.3 : $C^T(A^T + B^T) = C^T A^T + C^T B^T$

$\Rightarrow (C^T(A^T+B^T))^T = (C^T A^T + C^T B^T)^T$

$\Rightarrow (A^T+B^T)^T (C^T)^T = (C^T A^T)^T + (C^T B^T)^T$

$\Rightarrow ((A^T)^T + (B^T)^T) \cdot C = (A^T)^T \cdot (C^T)^T + (B^T)^T \cdot (C^T)^T$

$\Rightarrow \underline{(A+B) \cdot C = A \cdot C + B \cdot C}$.   (2)

Ex Lös matrisekvationen $\underset{(2/3)\cdot(3/2)}{A\,X} = \underset{(2/2)}{B}$, där

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 5 \end{pmatrix} \quad \text{och} \quad B = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}.$$

$$\overset{A}{\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 5 \end{pmatrix}} \overset{X}{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix}} = \overset{B}{\begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}}$$

$$\begin{array}{cc} x & y. \end{array} \qquad \begin{array}{cc} b_1 & b_2 \end{array}$$

Sätt: $X = (x \;\; y)$
$B = (b_1 \;\; b_2)$

$$A\,X = B \iff \begin{cases} Ax = b_1 & \longleftrightarrow \quad (A \mid b_1) \\ Ay = b_2 & \longleftrightarrow \quad (A \mid b_2) \end{cases}$$

Löser systemen med schemat: $(A \mid b_1\, b_2) = (A \mid B)$

$$\left(\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 2 \\ 2 & 0 & 5 & 5 & 3 \end{array}\right) \xrightarrow{BO1^{+}} \underset{(E)}{\left(\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 2 \\ 0 & -6 & 3 & 3 & -1 \end{array}\right)} \xrightarrow{BO3} \left(\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 2 \\ 0 & -3 & 3/2 & 3/2 & -1/2 \end{array}\right)$$

$$\xrightarrow[BO3]{BO1} \left(\begin{array}{ccc|cc} 1 & 0 & 5/2 & 5/2 & 3/2 \\ 0 & 1 & -1/2 & -1/2 & 1/6 \end{array}\right) \quad (RE)$$

Sätter: $x_3 = s$, $y_3 = t$, $s, t \in \mathbb{R}$.

$$\therefore \begin{cases} x_1 = \frac{5}{2} - \frac{5}{2}s \\ x_2 = -\frac{1}{2} + \frac{1}{2}s \\ x_3 = s \end{cases}, \qquad \begin{cases} y_1 = \frac{3}{2} - \frac{5}{2}t \\ y_2 = \frac{1}{6} + \frac{1}{2}t \\ y_3 = t \end{cases}$$

$$\therefore \; X = \begin{pmatrix} \frac{5-5s}{2} & \frac{3-5t}{2} \\ \frac{-1+s}{2} & \frac{1+3t}{6} \\ s & t \end{pmatrix}, \quad s, t \in \mathbb{R}.$$