$\boxed{\text{Ex}}$ (se 2(c), v. 17)

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -2 & 3 & -2 \\ 2 & 0 & 5 \end{pmatrix}, \qquad \underline{\lambda = \begin{cases} 3, \\ 5 \end{cases} \quad \text{Egenvärden}}.$$

$$\begin{cases} V(3) = \text{spn}\{ \overbrace{(0\ 1\ 0)^T}^{b_1}, \overbrace{(-1\ 0\ 1)^T}^{b_2} \} \\ V(5) = \text{spn}\{ \underbrace{(0\ -1\ 1)^T}_{b_3} \} \end{cases}$$

<u>Egenrum</u>.

$\{b_1, b_2, b_3\}$ <u>linjärt oberoende</u>. <u>Sätt</u>: $B = (b_1\ b_2\ b_3)$

$$\underline{B = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}}. \qquad B^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

<u>B diagonaliserar A</u>:

$$\underline{B^{-1}AB = D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}}$$

$$\Rightarrow \quad A = BDB^{-1} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\therefore \underline{\underline{A^n}} = BD^nB^{-1} = \underline{\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3^n & 0 & 0 \\ 0 & 3^n & 0 \\ 0 & 0 & 5^n \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}}$$

$$\left( A^2 = (BD\underbrace{B^{-1})(B}_{=I}DB^{-1}) = BD^2B^{-1} \quad \text{o.s.v} \right)$$

$$A^3 = (BD\underbrace{B^{-1})(B}_{=I}D\underbrace{B^{-1})(B}_{=I}DB^{-1}) = BD^3B^{-1}$$

$\vdots$

Ex] (20.5.08) $A = \begin{pmatrix} 5 & 2 & 5 \\ 2 & 17 & 2 \\ 5 & 2 & 5 \end{pmatrix}$.

Kolla att: $\lambda = \begin{cases} 0, \\ 9, \\ 18 \end{cases}$ och $\begin{cases} V(0) = \text{span}\left\{ \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{a_1} \right\} \\ V(9) = \text{span}\left\{ \underbrace{\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}}_{a_2} \right\} \\ V(18) = \text{span}\left\{ \underbrace{\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}}_{a_3} \right\} \end{cases}$

$a_1, a_2, a_3$ ortogonala, ty $A$ symmetrisk.

Sätt: $q_1 = \frac{a_1}{\|a_1\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, $q_2 = \frac{a_2}{\|a_2\|} = \frac{1}{3} \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $q_3 = \frac{a_3}{\|a_3\|} = \frac{1}{3\sqrt{2}} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$

Matrisen $Q = (q_1 \ q_2 \ q_3) = \begin{pmatrix} -1/\sqrt{2} & 2/3 & 1/3\sqrt{2} \\ 0 & -1/3 & 4/3\sqrt{2} \\ 1/\sqrt{2} & 2/3 & 1/3\sqrt{2} \end{pmatrix}$

ortogonal och diagonaliserar $A$:

$$Q^T A Q = D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 18 \end{pmatrix}.$$

$$\left( A = Q D Q^T, \quad A^n = Q D^n Q^T \right)$$

Ex $\begin{cases} p(t) = a_0 + a_1 t + \dots + a_r t^r. \\ P(A) = a_0 \cdot I + a_1 A + \dots + a_r A^r. \\ A = B D B^{-1} \quad (\text{diagonaliser bar}) \end{cases}$

$$p(A) = a_0 \cdot \overset{BB^{-1}}{\overset{\shortparallel}{I}} + a_1 B D B^{-1} + \dots + a_r B D^r B^{-1}$$
$$= B ( a_0 + a_1 \cdot D + \dots + a_r D^r ) \cdot B^{-1}$$
$$= B\, p(D)\, B^{-1}$$
$$= B \cdot \begin{pmatrix} p(\lambda_1) & & 0 \\ & \ddots & \\ 0 & & p(\lambda_n) \end{pmatrix} \cdot B^{-1}$$