Ex) $p = \dfrac{a^T b}{\|a\|^2} a \qquad (1)$

$$\underline{p^T \cdot (b-p)} = p^T b - p^T p = \frac{a^T \overbrace{b^T a}^{\in \mathbb{R},\ = a^T b}}{\|a\|^2} \cdot b - \overbrace{\frac{a^T b^T a}{\|a\|^2} \cdot \frac{a^T b a}{\|a\|^2}}^{= (a^T b)^2}$$

$$= \frac{(a^T b)^2}{\|a\|^2} - \left(\frac{a^T b}{\|a\|^2}\right)^2 \cdot \overbrace{\|a\|^2}^{= a^T a} \ \underline{= 0}.$$

Ex (20.5.08) Bestäm projektionen av 1°) a på b och 2°) b på a då
$a = (2\ 0\ 2)^T$ och $b = (0\ 3\ 3)^T$.

1°) $\underline{p} = \underset{(2)}{\dfrac{bb^T}{\|b\|^2}} a = \underset{(1)}{\dfrac{b^T a}{\|b\|^2}} b = \dfrac{(0\cdot 2 + 3\cdot 0 + 3\cdot 2)}{0^2 + 3^2 + 3^2} = \dfrac{6}{18}\begin{pmatrix}0\\3\\3\end{pmatrix} = \underline{\begin{pmatrix}0\\1\\1\end{pmatrix}}$

2°) $\underline{p} = \underset{(2)}{\dfrac{aa^T}{\|a\|^2}} b = \dfrac{a^T b}{\|a\|^2} \cdot a = \dfrac{6}{(2^2+2^2)} \cdot a = \dfrac{6}{8}\begin{pmatrix}2\\0\\2\end{pmatrix}$

$= \underline{\dfrac{3}{2} \cdot \begin{pmatrix}1\\0\\1\end{pmatrix}}.$

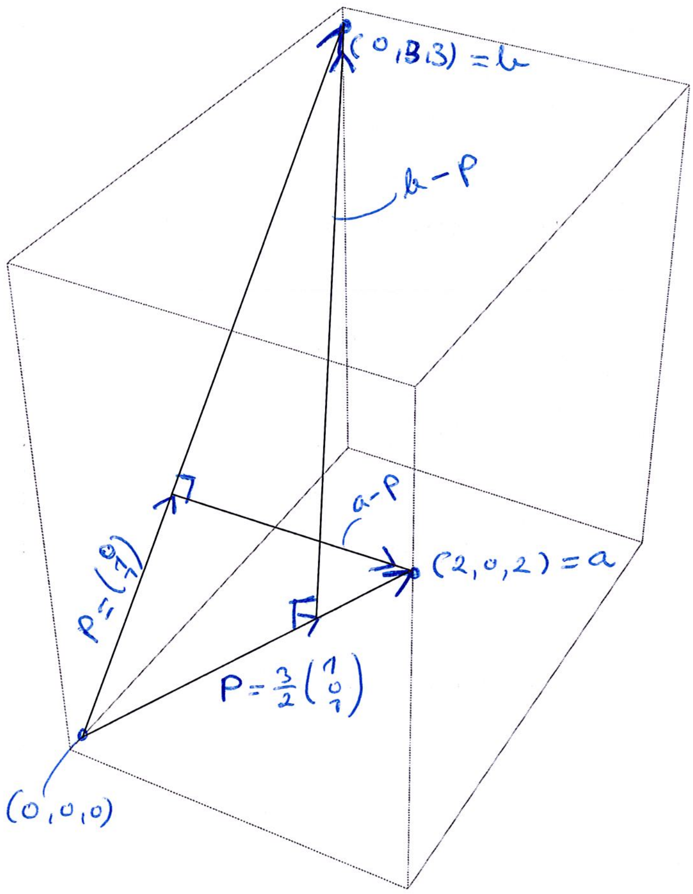
Ex) (20.5.08)

```
In[1]:= linea = Line[{{0, 0, 0}, {2, 0, 2}}]

Out[1]= Line[{{0, 0, 0}, {2, 0, 2}}]

In[2]:= lineb = Line[{{0, 0, 0}, {0, 3, 3}}]

Out[2]= Line[{{0, 0, 0}, {0, 3, 3}}]

In[3]:= linep1 = Line[{{2, 0, 2}, {0, 1, 1}}]

Out[3]= Line[{{2, 0, 2}, {0, 1, 1}}]

In[4]:= linep2 = Line[{{0, 3, 3}, {3 / 2, 0, 3 / 2}}]
```

Out[4]= $\text{Line}\left[\left\{\{0, 3, 3\}, \left\{\frac{3}{2}, 0, \frac{3}{2}\right\}\right\}\right]$

```
In[5]:= Show[Graphics3D[{linea, lineb, linep1, linep2}]]
```

Out[5]=



Ex] Bestäm minstakvadratlösningen till $Ax = b$, då

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}.$$

$$\underline{(A^T A \mid A^T b)} = A^T (A \mid b) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{pmatrix} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 2 & 0 & 3 & 4 \\ 0 & 6 & -3 & 2 \\ 3 & -3 & 6 & 5 \end{array}\right) \xrightarrow{BO} \cdots \longrightarrow \begin{array}{c} \begin{array}{cccc} & & F & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 3/2 & 2 \\ 0 & 1 & -1/2 & 1/3 \\ 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{array}{cccc} & & s & \end{array} \end{array}, \quad \begin{cases} x_1 = 2 - \frac{3}{2}s \\ x_2 = \frac{1}{3} + \frac{1}{2}s \\ x_3 = s \end{cases}$$

$$\therefore \quad x = \frac{1}{3}\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} + \frac{s}{2} \cdot \underbrace{\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}}_{} \in N(A^T A) = N(A)$$

Projektionen av b på $R(A)$ är:

$$\underline{\underline{p}} = Ax = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix} \left(\frac{1}{3}\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} + \frac{s}{2}\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \right)$$

$$= \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix}\begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix}$$

$$= \underline{\underline{\frac{1}{3}\begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}}}.$$

Ex (22.5.07)

```
In[1]:= lineb = Line[{{0, 0, 0}, {2, 2, 1}}]
```

Out[1]= Line[{{0, 0, 0}, {2, 2, 1}}]

```
In[52]:= plane = Line[{{0, 0, 0}, {3, 3, 0}, {3 / 2, -3 / 2, 3}, {0, 0, 0}}]
```

Out[52]= Line$\left[\left\{\{0, 0, 0\}, \{3, 3, 0\}, \left\{\frac{3}{2}, -\frac{3}{2}, 3\right\}, \{0, 0, 0\}\right\}\right]$

```
In[3]:= linep = Line[{{0, 0, 0}, {7 / 3, 5 / 3, 2 / 3}}]
```

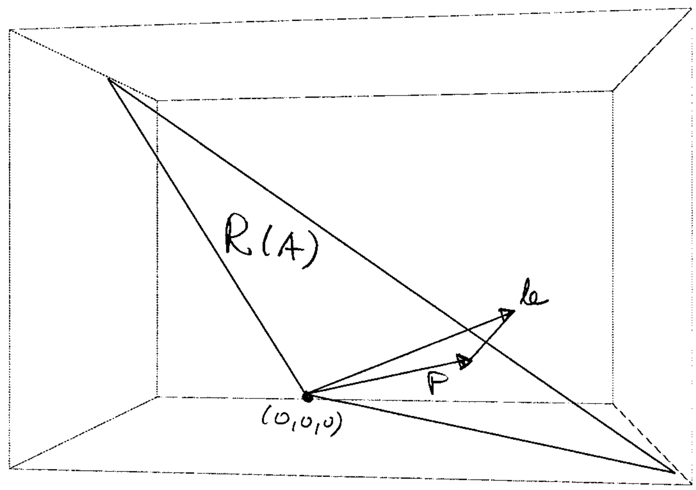
Out[3]= Line$\left[\left\{\{0, 0, 0\}, \left\{\frac{7}{3}, \frac{5}{3}, \frac{2}{3}\right\}\right\}\right]$

```
In[4]:= lineperp = Line[{{7 / 3, 5 / 3, 2 / 3}, {2, 2, 1}}]
```

Out[4]= Line$\left[\left\{\left\{\frac{7}{3}, \frac{5}{3}, \frac{2}{3}\right\}, \{2, 2, 1\}\right\}\right]$

```
In[53]:= Show[Graphics3D[{plane, lineb, linep, lineperp}], ViewPoint → {1.8 / 1.1, 0.2 / 1.1, 0}]
```

Out[53]=



$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -1 \end{pmatrix} \xrightarrow{\text{Bo1}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} R(A) = \text{Spn}\left\{ (1\ 1\ 0)^T, (1\ -1\ 2)^T \right\}. \\ b = (2\ 2\ 1)^T \notin R(A) \\ p = \frac{1}{3}\begin{pmatrix} 7 \\ 5 \\ 2 \end{pmatrix}, \quad \text{projeksjonen av } b \text{ på } R(A). \end{cases}$$

(186,5)

Ex 8.4. $U: \; x + 3\overset{F}{y} + \overset{F}{z} = 0 \quad i \; \mathbb{R}^3$

(använder (3) "direkt")

${}_{s} \quad {}_{t}$

plan.

$$\begin{cases} x = -3s - t \\ y = s \\ z = t \end{cases}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \cdot \underbrace{\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}}_{a_1} + t \cdot \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{a_2}$$

$U = \operatorname{spn}\{a_1, a_2\}$.

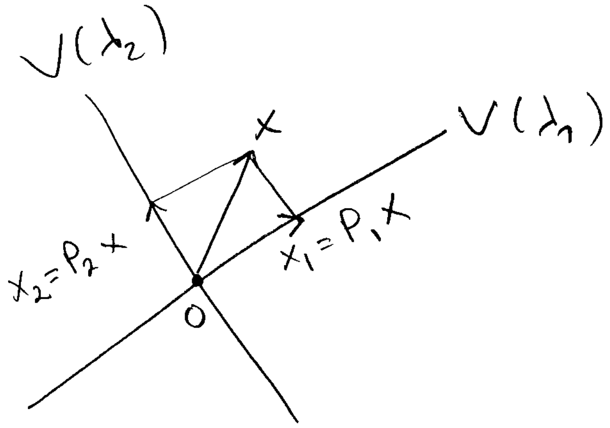
Sätt: $A = \begin{pmatrix} -3 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 10 & 3 \\ 3 & 2 \end{pmatrix}$$

$$(A^T A)^{-1} = \begin{pmatrix} 2/11 & -3/11 \\ -3/11 & 10/11 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 2 & -3 \\ -3 & 10 \end{pmatrix}$$

$$P = A\,(A^T A)^{-1} A^T = \begin{pmatrix} -3 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{11}\begin{pmatrix} 2 & -3 \\ -3 & 10 \end{pmatrix} \cdot \begin{pmatrix} -3 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{11}\begin{pmatrix} 10 & -3 & -1 \\ -3 & 2 & -3 \\ -1 & -3 & 10 \end{pmatrix}$$



$$X = x_1 + x_2$$

$$\forall\ x:\ (P_2 P_1) x = P_2 (P_1 x) = 0$$

$$\therefore\ \underline{P_2 P_1 = 0}\ .$$

$$\forall x:\ x = P_1 x + P_2 x = (P_1 + P_2) x$$

$$\therefore\ \underline{P_1 + P_2 = I}\ .$$

Ex| Existerar $\lim_{n\to+\infty} A^n$, då $A = \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix}$?

Kolla att: $\begin{cases} \lambda_1 = 1 \\ \lambda_2 = 1/2 \end{cases}$ och $\begin{cases} V(1) = \text{spn}\left\{ \overbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}^{=a_1} \right\} \\ V(1/2) = \text{spn}\left\{ \underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{=a_2} \right\} \end{cases}$.

Projektionsmatriserna på $V(1)$ och $V(1/2)$:

$$\begin{cases} P_1 = \dfrac{a_1 a_1^T}{\|a_1\|^2} = \dfrac{1}{2}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix} = \dfrac{1}{2}\cdot\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ P_2 = \dfrac{a_2 a_2^T}{\|a_2\|^2} = \dfrac{1}{2}\cdot\begin{pmatrix} -1 \\ 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \end{pmatrix} = \dfrac{1}{2}\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \end{cases}$$

Då gäller:

$$A^n = \lambda_1^n \cdot P_1 + \lambda_2^n \cdot P_2 = \underbrace{1^n}_{=1,\,\forall n} \cdot P_1 + \underbrace{\left(\tfrac{1}{2}\right)^n}_{\to 0,\text{ då } n\to+\infty} \cdot P_2$$

$$\longrightarrow P_1, \text{ då } n \to +\infty.$$

Svar: Ja, $\lim_{n\to+\infty} A^n = \dfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.