273032 Hilbert Spaces II

Home work #2, due 16.11.2010

MODIFIED VESION !

(40) Let H be a Hilbert space and let $T \in \mathcal{L}(H)$. Show that the operators

$$T^*T, \quad TT^*, \quad A = \tfrac{1}{2}(T+T^*), \quad B = \tfrac{1}{2i}(T-T^*)$$

are all self-adjoint, and that

$$T = A + iB, \quad T^* = A - iB.$$

Also show that $AB = BA$ if and only if $TT^* = T^*T$.

(An operator satisfying $T^*T = TT^*$ is called normal. We call A the real part of T and B the imaginary part of T. Sometimes iB is called the skew-adjoint part of T. An operator C is skew-adjoint if $C^* = -C$. Note that $(iB)^* = -iB$.

(41) Find the eigenvalues and the eigenfunctions for the following Sturm-Liouville systems:

(d) $x^2 f'' + 3xf' + \lambda f = 0, \qquad f(1) = 0 = f(e)$.

(42) 10.1. Show that an indefinite integral of an L^2 function is continuous. That is, if $f \in L^2(a, b)$ and $x_0 \in [a, b]$, then F defined by

$$F(x) = \int_{x_0}^{x} f(\tau)\, d\tau$$

is continuous on $[a, b]$. Show that F need not be differentiable on $[a, b]$ by taking f to be a step function.

description of all controllers K which will make the feedback system internally stable. This problem has a very elegant solution: it is elementary but far from obvious. Starting from G one can construct polynomial matrices A, B, C, D of appropriate types so that the desired controllers K are precisely those expressible in the form

$$K = (AQ + B)(CQ + D)^{-1} \tag{14.5}$$

for some rational matrix Q analytic in the right half plane. This is called the *Youla parametrization* (see Francis, 1986, Chapter 4). If it were just a matter of finding *some* internally stabilizing controller we could simply take $Q = 0$. In a real design problem, however, there will be many other factors to be taken into consideration. How well will the system stand up to unpredictable external disturbances (gusts of wind or a stewardess wheeling a drinks trolley down the aisle)? What will be the effect of the infinitesimal delays which are inevitable in a real physical construct? Presumably the exercise of control costs something: can we avoid undue extravagance? Clearly there is room for cunning in the choice of Q. This is no place to tackle all the complexities of a real engineering system: the purpose of this chapter will be served if we can appreciate how functional analysis enters into the treatment of one important problem: robust stabilization.

A mathematical model never describes the behaviour of a plant exactly. A controller which looks good on paper may turn out in practice to be so sensitive to small variations in model parameters or to deviations of the real system from the model (such as non-linearities) as to be useless. Some design methods are better than others in this respect, and engineers rely on their experience and intuition to select a design and on thorough testing to check its performance. The theory of feedback has been a relatively small part of the practice of control engineers. However, the march of technology is making for new requirements. As control systems have to perform more and more complicated tasks, intuition becomes harder to attain. It is difficult to develop a feel for a system with many inputs and outputs. Furthermore, it can happen that testing is impractical: a satellite which is designed to operate in zero gravity cannot be tested at the surface of the earth and has to work first time. To some extent testing the real thing can be replaced by computer simulation, but there is a clear need for a theory which takes account of the imprecise nature of mathematical models. This brings us to the notion of robustness. A design is *robust* if it works not only for the postulated model, but also for neighbouring models. A recent approach (since 1980) to formalizing this notion is to interpret closeness of models as closeness of their transfer functions in the H^∞-norm. There are