27 3031 Hilbert Spaces I

Last homework 12.10.2010 (note: Tuesday)

Last lecture on Tuesday, 5.10.2010

Solutions to last homework on Wednesday, 13/10.

Exam on October 15, 8.45–13.

31 7.34. Find the spectrum of the operator D of Problem 7.1, when (λ_n) is a bounded sequence.

32 7.37. Let $A \in \mathscr{L}(H)$, H a Hilbert space. Show that

$$\sigma(A^*) = \{\bar{\lambda} : \lambda \in \sigma(A)\}.$$

33 7.38. Find $\sigma(S)$ where S is the shift operator on ℓ^2 (Example 7.2(iv)).

and the equations can be solved to give

$$x(t) = Ae^{2t} + Be^{-3t} + (h * u)(t), \tag{14.4}$$

where $h * u$ is a particular integral of (14.3) and A, B are constants. It should be easy to accept that the term Ae^{2t} in (14.4) would be troublesome for a real system: presumably we do not want the state to go off to infinity as time passes. This term is present because the transfer function G has a pole at $s = 2$. Any pole of G in the right half plane $\{s \in \mathbb{C} : \operatorname{Re} s > 0\}$ will give rise to an exponential term in the solution for $x(\cdot)$ which tends to infinity with t. One of Maxwell's contributions was to identify stable systems as those whose transfer functions are bounded and analytic in the right half plane (actually the issue is a good deal more subtle, but this will do for present purposes).

One of the main points of the use of automatic controllers is to get an initially unstable system to behave stably. In the case of the system described by equation (14.4) this entails choosing $u(\cdot)$ so that the particular integral term $h * u$ counterbalances the destabilizing exponential term Ae^{2t}. This can be achieved by means of a 'feedback loop', which we can describe with the aid of a block diagram. The uncontrolled plant with transfer function G is represented by Diagram 14.1. This is simply another way of writing the equation $x = Gu$, which is equation (14.2) but for the fact that we are dispensing with the bars which indicate Laplace transforms. A feedback controller takes some measurement of the state function $x(\cdot)$, transforms it in some way and subtracts the result from the input to the system. If a human operator inputs a signal $u(t)$ and the state at time t is $x(t)$, then the input which actually reaches the system is $u(t) - (Kx)(t)$, where K is some operator which represents the action of the controlling device. This set-up can be shown by Diagram 14.2.

The engineer will naturally design the controller so that its action can be analysed, and this usually means that he chooses a controller which can be modelled by linear constant coefficient differential equations. Thus the controller will also be represented by a transfer function, and so we may regard all of u, x, G and K as functions of the complex variable s, of suitable (vector or matrix) types. Since the plant has input $u - Kx$ and state x we have

$$x = G(u - Kx)$$

Diagram 14.1