Grundkursen i sannolikhetslära 27.06.98, förslag till lösningar

2. $P(A \cap B | A \cup B) = \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)} = \frac{P(A\cap B)}{P(A\cup B)} = \frac{P(A\cap B)}{P(A)+P(B)-P(A\cap B)}$

   $= \frac{P(A)\cdot P(B)}{P(A)+P(B)-P(A)\cdot P(B)} = \frac{0.2}{0.9-0.2} = \frac{2}{7}$

4. $\xi \sim Po(1), f_\xi(k) = \frac{\lambda^k}{k!}e^{-\lambda} = \frac{1}{k!e}, k = 0, 1, 2, ...$

   $P(\xi < 2) = P(\xi = 0) + P(\xi = 1) = \frac{2}{e}$ $\quad \eta =$ ant. ind. med $\xi < 2$

   $\eta \sim Bi(6, \frac{2}{e}), f_\eta(k) = \binom{6}{k} (\frac{2}{e})^k (1-\frac{2}{e})^{6-k}, k = 0, 1, ..., 6.$

   Sökes $P(\eta \leq 1)$

   $P(\eta \leq 1) = P(\eta = 0) + P(\eta = 1) = (1-\frac{2}{e})^6 + 6 \cdot \frac{2}{e}(1-\frac{2}{e})^5.$

5. $\Omega_\xi = \{0, 1, 2\}$ Händelsen "$\xi = i$" betecknas k_i.

   $P(k_i) = P(J) \cdot P(k_i|J) + P(U) \cdot P(k_i|U) (*)$

   $P(J) = P(U) = \frac{1}{2}$

   $P(k_0|U) = \frac{1}{2}, P(k_1|U) = \frac{1}{2}, P(k_2|U) = 0$

   $P(k_0|J) = \binom{2}{0} (\frac{1}{2})^0 (\frac{1}{2})^2 = \frac{1}{4}$

   $P(k_1|J) = \binom{2}{1} (\frac{1}{2})^1 (\frac{1}{2})^1 = \frac{1}{2}$

   $P(k_2|J) = \binom{2}{2} (\frac{1}{2})^2 (\frac{1}{2})^0 = \frac{1}{4}$

   Beräknar nu alla $P(k_i), i = 0, 1, 2$ ur formeln $(*)$

   $P(k_0) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$

   $P(k_1) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

   $P(k_2) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot 0 = \frac{1}{8}.$

6. $\xi \sim Bi(100, \frac{1}{2})$

   $P(\xi = k) = \binom{100}{k} (\frac{1}{2})^k (\frac{1}{2})^{100-k} = \binom{100}{k} (\frac{1}{2})^{100}$

a) $P(\xi = 60) \approx 0.0108$

b) de M-L, se föreläsningen sid 64.