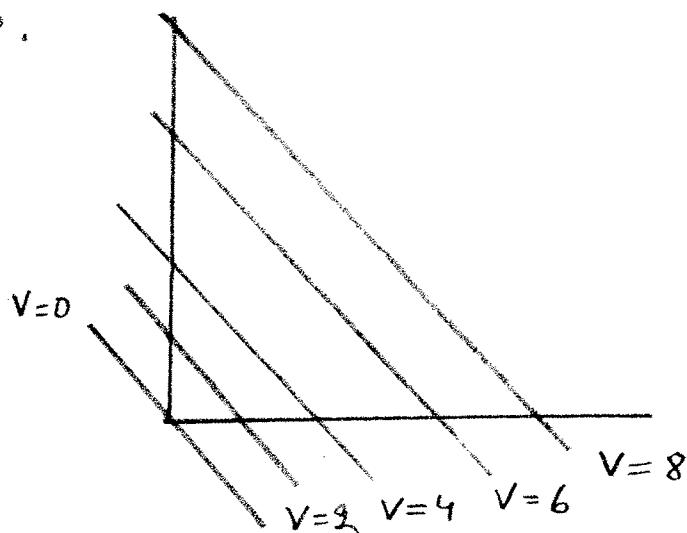


Ex.

Let us consider system from exercise 11.

Let $V = x+y$. The level curves are seen in the figure.



The derivative of V with resp. to the system is

$$\dot{V} = \dot{x} + \dot{y} = x(2-x-y) + y(6-2x-y) = 2x + 6y - x^2 - 3xy - y^2$$

In $x, y > 0$:

$$\text{If } x+y < 2 \text{ we have } \dot{V} = 2x + 6y - x^2 - 3xy - y^2 =$$

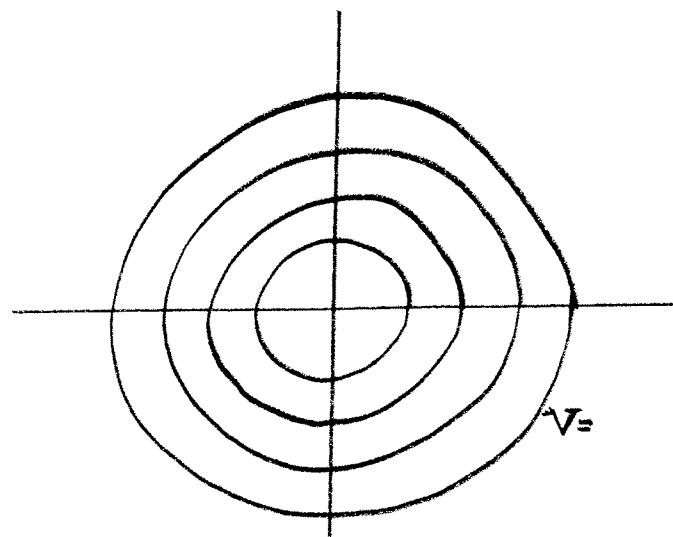
$= 2(x+y) - (x+y)^2 + (4-x)y > 0$ meaning that along trajectories V increases and we move up on the level curves.

If $x+y > 6$ we have $\dot{V} = 6(x+y) - (x+y)^2 - 4x - xy < 0$ meaning that V decreases along the trajectories and we go down on the level curves in that region.

Thus we come into the region $2x+y \leq 6$.

Compare this result with the phase portrait of the system.

Let now $V = x^2 + y^2$. The level curves are seen in the figure



The derivative of V is $\frac{V}{x} = 2x \cdot x(2-x-y) + 2y \cdot y(6-2x-y) = 2(2x^2 - x^3 - x^2y + 6y^2 - 2xy^2 - y^3)$

In $x, y \geq 0$

If $2x+y \leq 2$ we have $\frac{V}{x} = (2-y-2x)(x^2+y^2) + 4y^2 + x^3 > 0$, V increases and we move out from origo.

If $x+y > 6$ we have $\frac{V}{x} = 6x^2 + 6y^2 - (x+y)y - 4x^2 - x(x^2+y^2) - xy^2 < 0$, V decreases and we move towards origo.