

273028 Special Course in Dynamical
Systems

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Docent Gunnar Söderbacka

Hours: Mo 10-12 Tillsändorummets
Th 13-15 Ringborn
F 10-12 Tillsändorummets

Occasionally in Axelia PC-class

Starts Monday, October 26

Ends (prob.) by December 18

Course material on web page

web. abo.fi / fak / muf / mate /
kunser / dynsys

Introduction

The emphasis in this course will be on dynamical systems in continuous time, mathematically speaking, behavior of systems of ordinary differential equations.

In particular, we focus on disease modeling, epidemic models and malaria models.

$X(t)$ a vector in state space, usually a subset of \mathbb{R}^n (typically a quadrant or a half-space)

$X(t)$ evolves according to a differential equation

$$\frac{dX}{dt} = F(X(t), t)$$

Note: First-order eqn., because higher order eq. can be written as 1st order with larger state space.

2.

Ex. $S(t)$ "no." of susceptible individuals at time t
 $I(t)$ infected

$$\frac{dS(t)}{dt} = -\frac{\beta}{N} S(t) I(t)$$

$$\frac{dI(t)}{dt} = +\frac{\beta}{N} S(t) I(t)$$

$$I(0) + S(0) = N, \quad I(0) > 0.$$

Classification

linear if F is linear or affine *

Nonlinear

Autonomous if $\frac{dx}{dt} = F(x(t))$

Non-autonomous if $\frac{dx}{dt} = F(x(t), t)$

* F is of the form $A(t)X(t)$ or $A(t)X(t) + f(t)$, respectively, where $A(t)$ is a matrix of appropriate dim.

3. Goals: 1. Given

$$(*) \quad \frac{dx}{dt} = F(x(t), t), \quad x(0) = x_0 \in \mathbb{R}^n$$

a curve $x(t)$ satisfying $(*)$ is a solution.

What happens with $x(t)$, when $t \rightarrow \infty$

What happens with $x(t)$, when x_0 is perturbed
a little

2. If F depends on a parameter β , say,
what changes occur when β moves. Are
there dramatic shifts, "phase transitions"?

Often we have a constant solution $x(t) = C$.

Then $\frac{dx}{dt}(C) = 0$.

This is a fixed point of the system.

Ques.: If $x_0 \neq C$ but $x_0 = C + \varepsilon$ how does
the system behave. If

$x(t)$ remains bounded (for
all ε in a small nbhd of 0) then the
fixed point is stable.

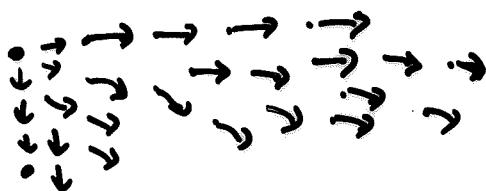
If $\lim_{t \rightarrow \infty} x(t) = C$ (for all ε in a
small nbhd of 0) then it is asymptotically
stable.

4. Methods

direction field (autonomous systems)

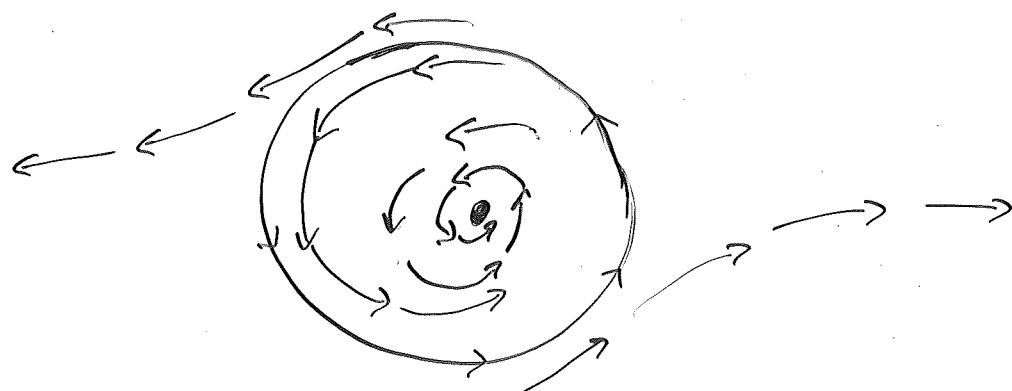
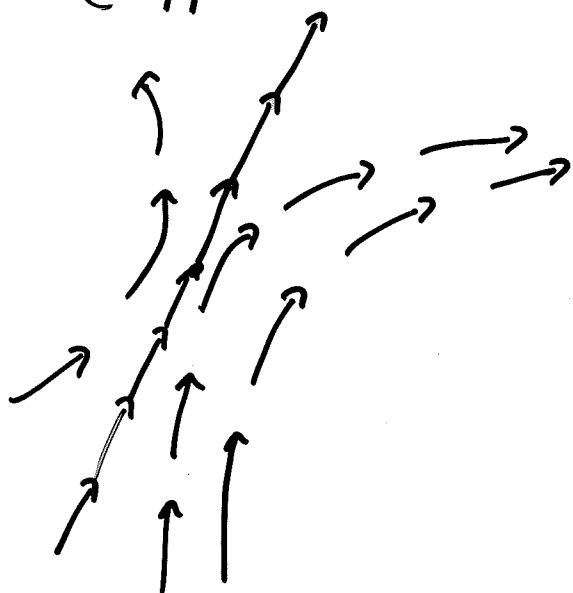
At a point X , draw a vector $F(X)$

In 2-D we can have, e.g.,



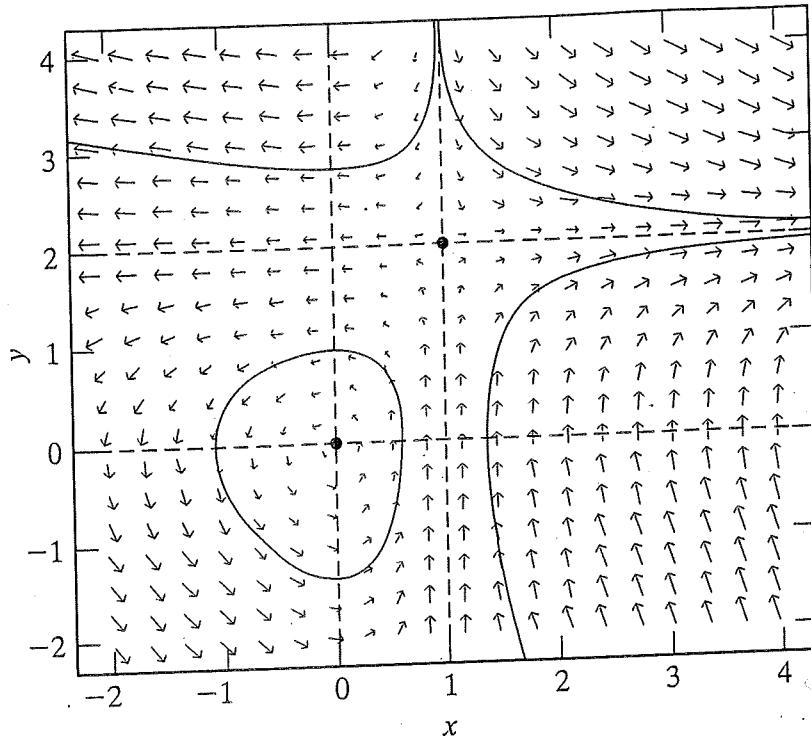
phase portrait

In the state space draw curves of solutions (approx.)



Nonlinear Ordinary Differential Equations: Theory and Examples

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Direction field of autonomous system

$$\frac{dx}{dt} = xy - y$$

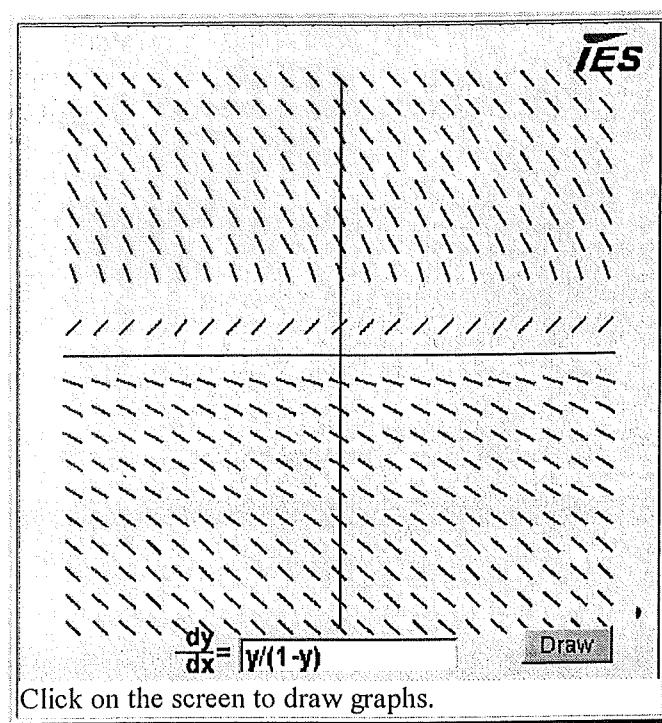
$$\frac{dy}{dt} = 2x - xy$$

Allen : Intro. to Math. Biology p. 194

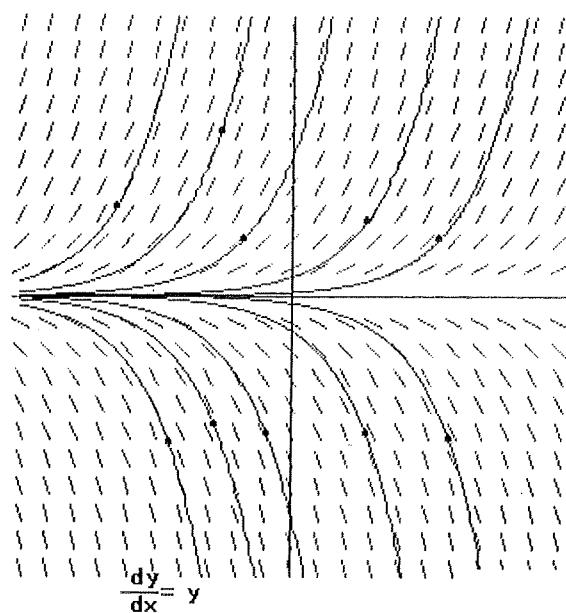
The First-Order Differential Equations

Direction Field of First Order Differential Equations

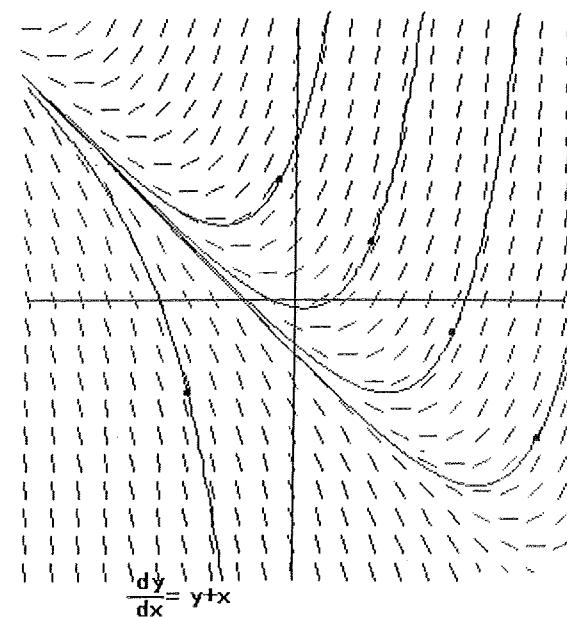
First-order differential Equation can be expressed in the form $dy/dx = f(x,y)$. The solution of the differential equation are certain functions. The differential equation defines the slope of at the point (x,y) of the certain curve of the function that passes through this point. For each point (x,y) , the differential equation defines a line segment with slope $f(x,y)$. We say that the differential equation defines the direction field of the differential equation.



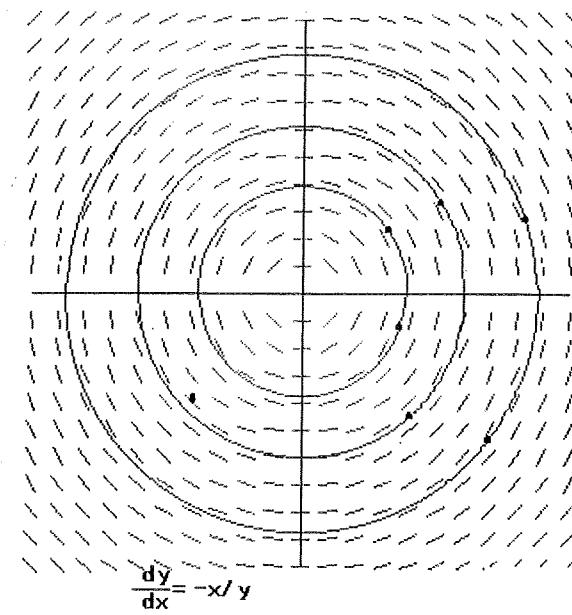
Examples



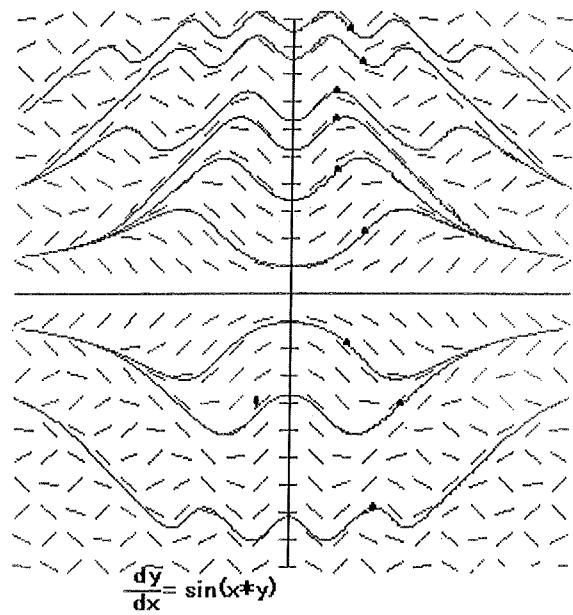
$$\frac{dy}{dx} = y$$



$$\frac{dy}{dx} = y+x$$



$$\frac{dy}{dx} = -x/y$$



$$\frac{dy}{dx} = \sin(x+y)$$