ANALYSIS II, Homework 1

Due Wednesday 18.9.2013. Please hand in written answers for credit.

1. The set of all real-valued polynomials with real coefficients and degree less or equal to n is denoted by $\mathcal{P}_n$. Show that $\mathcal{P}_n$ is a vector space over $\mathbb{R}$.

2. Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under scalar multiplication, but is not a linear subspace of $\mathbb{R}^2$.

3. Let E be a inner product space. Show that the following statements hold:

(a) If $x_1, ..., x_n \in E$ are such that $\langle x_i, x_j \rangle = 0$ for $i \neq j$, then

$$||\sum_{k=1}^{n} x_k||^2 = \sum_{k=1}^{n} ||x_k||^2.$$

(b) $||x+y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2$ for all $x, y \in E$.

4. Let E be a complex inner product space. Show that the following statements are valid for all $x, y, z \in E$:

(a) If $\langle x, y \rangle = \langle x, z \rangle$ for all $x \in E$, then $y = z$.

(b) $4\langle x, y \rangle = ||x+y||^2 - ||x-y||^2 + i||x+iy||^2 - i||x-iy||^2$.

5. If $f \in C([a,b], \mathbb{K})$, let $||f||_1 = \int_a^b |f(x)| \, dx$. Show that $|| \cdot ||_1$ is a norm in $C([a,b], \mathbb{K})$.

6. The space $(C([0, \frac{\pi}{2}], \mathbb{K}), || \cdot ||_\infty)$, where $||f||_\infty = \sup_{t \in [0, \frac{\pi}{2}]} |f(t)|$, is a normed space. Show that $(C([0, \frac{\pi}{2}], \mathbb{K}), || \cdot ||_\infty)$ is not an inner product space.