Exercise 9. Continue with the two-dimensional target density from Exercises 7 & 8:

$$f(x, y) \propto \exp(-2x^3y^3), x \in [-1, 2], y \in [-1, 2]. \tag{1}$$

Use the Gibbs sampler algorithm (course material Chapter 7) to generate samples from this density, i.e. each component of the vector (x, y) is updated using a separate proposal from the correct conditional density given the current value of the other component. This can be implemented, e.g., by using the naive rejection or more efficient rejection samplers that were considered in the first exercises. In this case, the rejection sampler is executed in a **while** loop, until an acceptable value is produced for the Gibbs step. Notice that the conditional density of x given y_t can be written as:

$$f(x|y_t) \propto \frac{\exp(-2x^3y_t^3)}{\int_{-1}^{2} \exp(-2x^3y_t^3)dx}, \tag{2}$$

and the conditional density of y given x_t as:

$$f(y|x_t) \propto \frac{\exp(-2x_t^3y^3)}{\int_{-1}^{2} \exp(-2x_t^3y^3)dy}. \tag{3}$$

When the proposal value is generated from such a density in the Metropolis-Hastings algorithm, it will always be accepted (the four densities cancel out in formula 7.7 and the ratio will be equal to unity). As $f(x|y_t)$ is a one-dimensional density with *parameter* $-2y_t^3$, the rejection algorithm investigated earlier can be used to produce a single sample from the corresponding distribution on $[-1, 2]$, which will then be accepted as the Gibbs update of x. Similar procedure obvisouly applies to $f(y|x_t)$.

Compare the Gibbs sampler behavior to that obtained in the previous two exercises with respect to the estimated properties of the underlying density (means, variances, covariance etc).