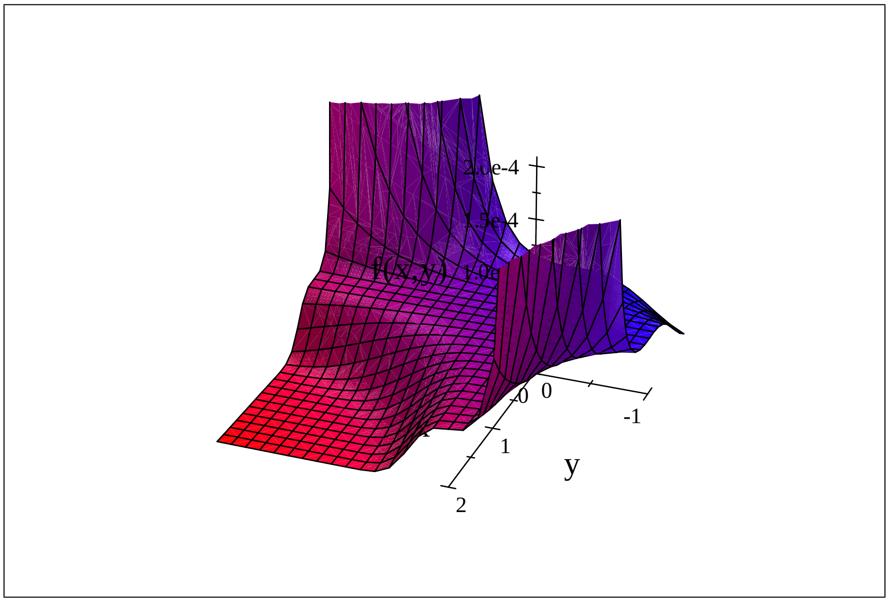
Exercise 7. Consider the following two-dimensional target density

$$f(x, y) \propto \exp(-2x^3y^3), x \in [-1, 2], y \in [-1, 2]. \quad (1)$$

Surface of the normalized density is shown below.



Use the following versions of a Metropolis-Hastings algorithm (course material Chapter 7) to generate samples from this density:

1. Independence sampler, where the proposal distribution is the uniform distribution on the rectangle $[-1, 2] \times [-1, 2]$ for any value of the current state (x_t, x_y) in the Markov chain.

2. Sampler with a symmetric proposal, equal to the bivariate Normal distribution centered at the current state (x_t, x_y) in the Markov chain, and with the covariance matrix $\sum = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$, where σ^2 can take different values, e.g. try out 1, 0.5 and 0.1.

3. Sampler with a symmetric proposal, equal to the bivariate Normal distribution centered at the current state (x_t, x_y) in the Markov chain, and with the covariance matrix $\sum = \begin{pmatrix} \sigma^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma^2 \end{pmatrix}$, where σ^2 and σ_{xy} can take different values, e.g. try out σ^2 among the values 1, 0.5 and 0.1, and scale the covariance such that the correlation between X and Y in the proposal is either -0.5 or 0.5 (try both values).

Report for all cases the sampler behavior (acceptance rates of proposals, visualizations of sample trajectories etc). Compare the estimates of means, variances and covariance to those obtained by numerical integration using (1). Investigate how the level of Monte Carlo error decreases when the sample size increases.