

Exercise 4. Here we continue with the same target density as considered in the previous exercises, and approximate the target by unbiased importance sampling (Section 4.5.1.). Assume that the density  $g$  is the Normal(-1,1)-distribution truncated to the interval  $[-1,2]$ . Using the previously discussed truncation method, generate  $n$  samples  $x_1, \dots, x_n$  from  $g$ . Consider then behavior of the importance weights defined in formula (4.12)

$$w_i = \frac{f(x_i)}{g(x_i)}$$

and the corresponding estimate of  $h(X) = X^2$ , as a function of  $n$ , e.g. for the range  $n = 100, 500, 1000, 5000, 10000$ , of values. In particular, draw histograms for the obtained empirical distributions of the importance weights. Notice that here we assume that the normalizing constant of  $f$  (as well as of  $g$ ) is known. Numerical integration yields

$$\int_{-1}^2 \exp(-2x^3) dx = 2.7019.$$