Assignments II

T7. You have a sample of size n from a normal distribution $N(\mu, \sigma^2)$, with known variance σ^2. What is Jeffreys' prior for mean μ? If you assume μ known and variance unknown, what is Jeffreys' prior for the variance? Compare Jeffreys' priors which you get if (1) you assume both parameters to be unknown and use the formula, (2) you assume both parameters to be unknown, but not the location parameter μ to be fixed.

T8. Consider first the thumbtack tossing scenario discussed in the lecture materials. Derive the Jeffreys' prior for the probability that the thumbtack will land with the pin pointing down, when we make n tosses. Secondly, alter now the tossing scenario and count the number of tosses required until we have observed m tosses with the pin pointing down. What is the Jeffreys' prior in this tossing scenario? Consider how the sampling mechanism affects the shape of the prior.

T9. The first six pages of a draft version of the course material contained the following number of typos

$$3, 4, 2, 1, 2, 3.$$

How would your predictive distribution for the number of typos on page seven look like if you model the number of typos as exchangeable random quantities following a Poisson distribution with a Gamma prior for the rate parameter?

T10. The following problem is generally known as the Monty Hall problem and we consider also two other variants of it. Pay attention to the difference in the information content of the observations you make in the two situations, i.e. this mimics how the likelihood of data changes your conditional probabilities depending on how much your observation is worth.

You are in a quiz, where you have to choose one of three doors. Behind two of them there is a goat and the final door hides a sports car. After you have chosen a door (say A), the MC opens one of the other doors (B,C) and uncovers a goat (he doesn't cheat you). Now he asks you whether you would like to switch your door to the remaining one. Show explicitly that you improve your chances of getting the car by making the change? What happens if the situation is otherwise the same, except that there are 200 doors, with one car and 199 goats behind them, and the MC reveals either a single goat (by opening one door), or reveals 198 goats (by opening 198 doors)?

T11. You observe one real number x and wish to compare standard normal distribution to the standard Cauchy distribution. The density function of the latter equals

$$p(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$$

How does the Bayes factor behave as a function of x?