Definition 3.7 ***Erlang Random Variable***

X is an ***Erlang*** *(n, λ) random variable if the PDF of X is*

$$f_X(x) = \begin{cases} \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

where the parameter $\lambda > 0$, and the parameter $n \geq 1$ is an integer.

3.4.11 Calculate the kth moment $E[X^k]$ of an Erlang (n, λ) random variable X.

4.2.7 Each test of an integrated circuit produces an acceptable circuit with probability p, independent of the outcome of the test of any other circuit. In testing n circuits, let K denote the number of circuits rejected and let X denote the number of acceptable circuits (either 0 or 1) in the last test. Find the joint PMF $P_{K,X}(k, x)$.

4.5.6 Random variables X and Y have the joint PDF

$$f_{X,Y}(x, y) = \begin{cases} cy & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Draw the region of nonzero probability.

(b) What is the value of the constant c?

(c) What is $F_X(x)$?

(d) What is $F_Y(y)$?

(e) What is $P[Y \leq X/2]$?

4.7.9 Random variables X and Y have joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 4xy & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What are $E[X]$ and $\text{Var}[X]$?

(b) What are $E[Y]$ and $\text{Var}[Y]$?

(c) What is $\text{Cov}[X, Y]$?

(d) What is $E[X + Y]$?

(e) What is $\text{Var}[X + Y]$?

5.5.6 Let $X_1, \ldots, X_n$ denote n iid random variables with PDF $f_X(x)$ and CDF $F_X(x)$. What is the probability $P[X_n = \max\{X_1, \ldots, X_n\}]$?

6.6.1 The waiting time W for accessing one record from a computer database is a random variable uniformly distributed between 0 and 10 milliseconds. The read time R (for moving the information from the disk to main memory) is 3 milliseconds. The random variable X milliseconds is the total access time (waiting time + read time) to get one block of information from the disk. Before performing a certain task, the computer must access 12 different blocks of information from the disk. (Access times for different blocks are independent of one another.) The total access time for all the information is a random variable A milliseconds.

(a) What is $E[X]$, the expected value of the access time?

(b) What is $\mathrm{Var}[X]$, the variance of the access time?

(c) What is $E[A]$, the expected value of the total access time?

(d) What is σ_A, the standard deviation of the total access time?

(e) Use the central limit theorem to estimate $P[A > 116\text{ms}]$, the probability that the total access time exceeds 116 ms.

(f) Use the central limit theorem to estimate $P[A < 86\text{ms}]$, the probability that the total access time is less than 86 ms.