

**1.5.6** Deer ticks can carry both Lyme disease and human granulocytic ehrlichiosis (HGE). In a study of ticks in the Midwest, it was found that 16% carried Lyme disease, 10% had HGE, and that 10% of the ticks that had either Lyme disease or HGE carried both diseases.

- What is the probability  $P[LH]$  that a tick carries both Lyme disease ( $L$ ) and HGE ( $H$ )?
- What is the conditional probability that a tick has HGE given that it has Lyme disease?

**1.6.3** In an experiment,  $A$ ,  $B$ ,  $C$ , and  $D$  are events with probabilities  $P[A] = 1/4$ ,  $P[B] = 1/8$ ,  $P[C] = 5/8$ , and  $P[D] = 3/8$ . Furthermore,  $A$  and  $B$  are disjoint, while  $C$  and  $D$  are independent.

- Find  $P[A \cap B]$ ,  $P[A \cup B]$ ,  $P[A \cap B^c]$ , and  $P[A \cup B^c]$ .
- Are  $A$  and  $B$  independent?
- Find  $P[C \cap D]$ ,  $P[C \cap D^c]$ , and  $P[C^c \cap D^c]$ .
- Are  $C^c$  and  $D^c$  independent?

**1.6.7** For independent events  $A$  and  $B$ , prove that

- $A$  and  $B^c$  are independent.
- $A^c$  and  $B$  are independent.
- $A^c$  and  $B^c$  are independent.

**1.7.5** Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time. What is  $P[-|H]$ , the conditional probability that a person tests negative given that the person does have the HIV virus? What is  $P[H|+]$ , the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?

**2.5.10** Let binomial random variable  $X_n$  denote the number of successes in  $n$  Bernoulli trials with success probability  $p$ . Prove that  $E[X_n] = np$ .

**2.8.11** Random variable  $K$  has a Poisson ( $\alpha$ ) distribution. Derive the properties  $E[K] = \text{Var}[K] = \alpha$ .