On the power of an asymptotically optimal test for the case of Laplace distribution

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We present a result concerning a formula (see (3.1) in [1])

$$r(t) \equiv \lim_{n\to\infty} n^{-1/2}(\beta_n^*(t) - \beta_n(t)) = \frac{t^2}{3}\varphi(u_\alpha - t), \quad 0 < t \le C, \quad C > 0,$$

for the limit of the difference between the power of the asymptotically optimal test, $\beta_n(t)$, and the power of the asymptotically most powerful (AMP) test, $\beta_n^*(t)$, for the case of Laplace distribution, where $u_\alpha = \Phi^{-1}(1-\alpha)$, $\alpha \in (0,1)$, $\Phi(x)$ and $\varphi(x)$ denote the standard normal distribution function and the standard normal density, respectively. This difference has the order $n^{-1/2}$ due to the nonregularity of the Laplace distribution, in contrast to regular laws for which this order equals n^{-1}. The validity of the formula is proved by asymptotic expansions for the powers of the tests (see Theorems 2.1, 3.3 in [2]). And the general form of this formula (see Theorem 3.2.1 in [3]) was obtained with sufficient conditions related to the case of Laplace distribution. Since the difference relates to the asymptotic deficiency of a test, this problem may suggest an idea about the deficiency for a general nonregular case.

References

[1] Korolev R. A., Testova A. V., Bening V. E., On power of asymptotically optimal test for the case of Laplace distribution. *Vestnik Tverskogo gos. univer.*, series *Priklad. matematika* **8** (2008), no. 4(64), 5–23.

[2] Korolev R. A., Bening V. E., Asymptotic expansions for power of tests for the case of Laplace distribution. *Vestnik Tverskogo gos. univer.*, series *Priklad. matematika* **3(10)** (2008), no. 26(86), 97–107.

[3] Bening V. E., *Asymptotic Theory of Testing Statistical Hypotheses.* VSP, Utrecht, 2000.