A Sojourn Time Problem for Brownian Motion with Drift

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Abstract. Let γ^μ denote the total amount of time that a Brownian motion with drift spends above zero on the interval $[0, T]$. We consider the optimal prediction problem

$$V = \inf_{0 \leq \tau \leq T} \mathbb{E} \, |\gamma^\mu - \tau \, |$$

where the infimum is taken over all stopping times for the Brownian motion with drift. It turns out that this problem is rather more complicated than might be expected, and requires some extensions of Peskir's change of variable formula with local time on curves/surfaces. We present these extensions, as well as the solution to the optimal prediction problem.

Keywords optimal prediction; local time–space calculus; non–linear integral equations; free–boundary problems