PRIMITIVE CASINOS IN THE PRESENCE OF INFLATION

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Mr. G owes $100 000 to a loan shark, and will be killed at dawn if the loan is not repaid in full. Mr. G has $20 000, but partial payments are not accepted, and he has no other source of income or credit. The loan shark owns a primitive casino where one can stake any amount in ones possession, gaining r times the stake with probability w and losing the stake with probability $1 - w$ ($r > 0, 0 < w < 1$). Mr. G is permitted to gamble at the casino, but each time he places a bet, the amount of his debt is increased by a factor of $1 + \alpha$ ($\alpha \geq 0$). How should Mr. G gamble to maximize his chance of reaching his (moving) target and thereby surviving? Dubins and Savage showed that an optimal strategy is to stake boldly if the primitive casino is subfair or fair (i.e. $w(1 + r) \leq 1$) and the inflation rate α is 0. Intuitively, a positive inflation rate would motivate Mr. G to try to reach his goal as quickly as possible, so it seems plausible that the bold strategy is optimal. However, Chen, Shepp, and Zame found that, surprisingly, the bold strategy is no longer optimal for subfair primitive casinos with inflation if both $r > 1$ and α satises $1/r \leq \alpha < r$. They also conjectured that the bold strategy is optimal for subfair primitive casinos with inflation if $r < 1$. We will show that this conjecture is true provided that $w \leq 1/2$. We will also discuss Vardi's version of the problem where Mr. G is allowed to pick among a spectrum of games (r, w). This talk is based on joint work with Robert Chen, Larry Shepp and Yi-Ching Yao.