π options

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Abstract. We consider a discretionary stopping problem that arises in the context of pricing a class of perpetual American-type call options, which include the perpetual American, Russian and lookback-American call options as special cases. We solve this genuinely two-dimensional optimal stopping problem by means of an explicit construction of its value function. In particular, we fully characterise the free boundary that provides the optimal strategy, and which involves the analysis of a highly non-linear ordinary differential equation (ODE). It turns out that the associated variational inequality has uncountable solutions that satisfy the so-called "principle of smooth fit", and that identifying the value function relies on the so-called transversality condition in a rather non-trivial way.

Keywords American-type call options; variational inequality; free boundary; transversality condition