Minimising the Time to a Majority Decision

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Abstract. We consider a stochastic control problem in which the controller seeks to minimise the time taken for a Brownian populace to reach a majority decision. More specifically, suppose that we are given three independent Brownian Motions on the unit interval with absorption at the endpoints. At each instant, one must choose which of the three Brownian motions to run, the objective being to minimise the time taken for at least two to be absorbed at the same value. This model is motivated by a conjecture of Peres on a related discrete problem.

We formulate the problem in terms of optional increasing paths and propose a candidate optimal strategy. Then, using a heuristic argument, we compute the value function $\sup_c \mathbb{E}_x(e^{-r\tau_c})$ (where $r > 0$ and τ_c the decision time for control c). We conclude by deducing *stochastic minimality* of the decision time for our strategy.

Keywords switching; time allocation; majority vote; total positivity; stochastic minimum; recursive majority of three