# The Optimal Stopping of "Seasonal"Observations and the Game of "Seasonal"Stopping.

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**Abstract.** The following problem of optimal stopping (OS) of Markov chain (MC) was dubbed previously by an author as OS of "Seasonal" Observations. A decision maker (DM) observes a MC $(Z_n)$ with two components, where the first one is an "underlying" finite MC with $m$ states and given transition matrix $Q$, and the other component is one of $m$ independent sequences of i.i.d. r.v. with known distributions. A simple example of such a situation is a case in which there are two dice with four and six equally likely sides and the determination as to which of them is tossed at a given moment is specified by the position of a Markov chain with two states. The goal of a decision maker (player) is to maximize the discounted expected reward over all possible stopping times. The optimal stopping set is defined by a set of thresholds. We describe an algorithmic solution to their calculation. Some of these results were obtained in a joint (with E. Presman) paper submitted to Theory of Probability and its Appl. An important part of the solution is played by the Elimination Algorithm for the Problem of Optimal Stopping of Markov chain developed by the author earlier.

The Game of Seasonal stopping is a game where in a similar setting there are $N$ players, $N \leq m$, and a state space partitioned into $N$ nonempty subsets. A player $i$ can stop the game only when the underlying MC is in her "favorable" subset. Correspondingly, the policy of each player is defined by her stopping set $S_i$. The game stops at the moment of first visit of the MC $(Z_n)$ to the union of these stopping sets. Under mild restrictions on reward functions, we prove the existence of at least one Nash equilibrium and describe cases when there are multiple equilibria. An open problem is to describe all such equilibria.

**Keywords** Markov chain; optimal stopping; Elimination Algorithm; Nash equilibrium