Numerical method for optimal stopping of piecewise deterministic Markov processes

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Abstract. The aim of this paper is to propose a computational method for optimal stopping of a piecewise deterministic Markov process by using a quantization technique for an underlying discrete-time Markov chain related to the continuous-time process and path-adapted time discretization grids.

Piecewise-deterministic Markov processes (PDMP's) have been introduced in the literature by M.H.A. Davis (2) as a general class of stochastic models covering a great number of problems of operations research. PDMP's are a family of Markov processes involving deterministic motion punctuated by random jumps. It was shown in (3) that the value function of the optimal stopping problem can be calculated by iterating a functional operator which involves a continuous-time maximization and a discrete-time dynamic programming formula. To approximate the value function of the optimal stopping problem of a PDMP, a natural approach would have been to follow the same lines as in (1) for diffusion processes, unfortunately their method cannot be directly applied here. However, by using the special structure of PDMP's, we are able to propose an approximation scheme for the value function and derive a rate of convergence.

In addition and more importantly, this numerical approximation scheme enables us to propose, without any additional calculations, a computable stopping rule which also is an ϵ-optimal stopping time of the original problem, and we can characterize how far it is from optimal in terms of the value function.

Keywords piecewise deterministic Markov processes; quantization; numerical method; dynamic programming

References

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