# Modification of Sonin's algorithm for optimal stopping of Markov chain

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**Abstract.** The problem of optimal stopping of a Markov chain with an infinite horizon is considered. A general approach for finding the value function is to find recurrently the value function in the problem with not more than $k$ steps and to take a limit as $k \to \infty$. This procedure is considered as a constructive method for finding the value function, but even for the case of two states the exact value of the value function is obtained only in infinite number of steps.

Sonin (see [1]–[2]) for the case of finite number $m$ of states proposed an algorithm which allows one to find the value function and the stopping set in no more than $2m$ steps. This algorithm on each step considers a new chain with a new state space, eliminating the states where for the previous chain the reward of an immediate stopping is less than the reward of making one more step. In [3] some situations with countable state space were considered.

We discuss the case of an arbitrary state space, which requires a modification of the algorithm. On each step we consider a new chain with the same state space and the same initial state. The state of the new chain at time $n$ coincides with the state of the previous chain at the time of the $n$-th hitting of the set where for the previous chain the reward of stopping is greater or equal to the reward of making one more step. This algorithm allows one to give an independent proof of the theorem describing the properties of the value function and the optimal stopping time. A preliminary results were published in [4].

As an example we examine the simple random walk on the interval $[0, 32]$ with absorbing points 0 and 32. The payoff function $g(x)$ has local maxima $g(2) = 11,\ g(15) = 8,\ g(30) = 14$, local minima $g(8) = 4,\ g(22) = 5$, is concave at points $1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 18,\ 28, 29, 30, 31$, and convex at the other points. The standard recurrent procedure of finding the value function gives an approximation error of $5,84$ percent after 350 iteration. The proposed algorithm gives the exact value function after five steps.

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