Properties of a Maintenance System on a Partially Observable Markov Process and an Optimal Stopping Problem

Tōru Nakai
Faculty of Education, Chiba University
t-nakai@faculty.chiba-u.jp

Abstract. A sequential decision problem on a Markov process in which states of this process are closely related to an outcome is treated in Nakai (Recent Advances in Stochastic Operations Research II, World Scientific, 2009). In this problem, each state can be changed by expending an additional amount, and it also changes according to a Markovian transition rule based on the total positivity of order two. In this paper, we will consider a sequential decision problem to minimize the total expected cost and an optimal stopping problem.

We develop a sequential decision problem on a partially observable Markov process for the optimal maintenance policy for the products such as the electrical devices, the cars and so on. During their life cycle, conditions of these items changes and this condition causes some troubles. For a small trouble, it is possible to handle individually, but it is necessary to recall for a defective or faulty item. The decision-maker does not observe a condition of each item directly, but information is obtained through a magnitude of a claim from a consumer. For this purpose, conditions of an item are considered as a state of a Markov process with state space $(0, \infty)$. These states change according to a Markovian transition rule based on the total positivity of order two, which plays an important role in the Bayesian learning procedure for a partially observable Markov process. In this case, as $s \in (0, \infty)$ approaches to 0, this item complied with the user, and it is not sufficiently complied with their demands as s becomes greater. As for information about a unobservable state, a magnitude of a claim of a consumer is a random variable X_s for each state s. After observing this value, the decision-maker improves information about the current state of the process by employing the Bayesian learning procedure. All information is summarized by a probability distribution on the state space such as a log-normal distribution on it. On the other hand, the decision-maker decides whether to deal with each claim with a small cost depending on the magnitude or to recall this item with a large cost which varies with a decision. This problem is how much to expend to maintain this item, and the objective of this problem to minimize the total expected cost. The dynamic programming formulation implies a recursive equation about the expected cost obtainable under the optimal policy, and we observe monotonic properties for this value.

Keywords Partially Observable Markov Process; Sequential Decision Problem; Bayesian Learning Procedure;