Optimal Stopping with Two Types of Constraints

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Abstract. Let $X_1, X_2, \ldots, X_n$ be i.i.d. positive random variables having absolute continuous distribution function F. We observe the X_k's sequentially and must decide at each time instant k whether to select X_k or else to refuse and continue. The number of selections is subject to two concurrent constraints, each of a different type. On the one hand, we must select at least r of the X_k's; on the other hand, our selection strategy must also accept, *in expectation*, a minimum number of μ of the X_k's. We interpret X_i as the cost of item i. The objective is to minimize the expected total cost of selected items under the two constraints over the set of all non-anticipative online selection strategies.

In the terminology of optimal stopping problems, this problem is a finite-horizon i.i.d. full-information stopping problem. Here *full-information* refers to the hypothesis that the distribution F of the X_k's is known to us and that we are allowed to observe the actual *values* of these (and not only some statistic of these as for instance their rank). We add the description *with two types of constraints.*

As far as the author is aware, this combination of constraints is new. His interest in this problem was instigated by a side-question of David Aldous who formulated a problem of improving a lower bound (of interest in the context of spanning trees) as a stopping problem. His problem is the problem with definite constraint $r = 1$ and expectational constraint $\mu = 2$. The motivation is also going further. Indeed, the proposed mixed-constraint setting offers a variety of applications: In certain buying/selling problems, for instance, some contract specifications require not only a minimum number of purchases, but also a long-run average "chiffres d'affaires". In the case of buying, our proposed model fits the problem directly; in the case of selling we re-interpret the costs as benefits resulting from sales and would like to solve the maximization problem, which is equivalent. Another example of selection problems where this setting is relevant is the online-knapsack problem: Items, each having utility one, arrive in random order. Time constraints force us to select online from a sequence of incoming different items and to pack them without recall. Costs are measured in terms of weight or space, and the total combined utility is supposed to be at least r, with an average of at least μ.

Keywords selection problems; concurrent constraints; optimality principle; recursive functions; Riccati differential equation; minimal spanning tree