A Variational Approach To An Optimal Stopping And Free-Boundary Problems

Vadim Arkin
Alexander Slastnikov
Central Economics and Mathematics Institute, Moscow, Russia
arkin@cemi.rssi.ru, varkin@mail.ru

Abstract. We describe a variational approach to the solution to optimal stopping problems for multi-dimensional diffusion processes, as an alternate to the traditional approach based on the solution of the Stefan (free-boundary) problem. The connection of this variational approach to smooth pasting conditions is established. We present the examples where the solution to the Stefan problem is not the solution to an optimal stopping problem. In particular, the Stefan problem has a solution, whereas optimal stopping problem has no solution. On the base of the proposed approach, we obtain the solution to an optimal stopping problem for a two-dimensional geometric Brownian motion with a nonlinear payoff function.

Keywords Multi-dimensional diffusion processes; Stefan problem; variational approach; smooth pasting