Optimal Stopping Problem with Uncertain Stopping and its Application to Discrete Options

Katsunori Ano
Institute of Applied Mathematics
kano@iapm.jp

February 12, 2009

Abstract. We study the optimal stopping problem for the stationary Markov sequence, where the stopping offer is accepted with probability $p = 1 - q$ $(0 \leq p \leq 1)$. We are allowed to make m $(m \geq 1)$ stopping offers until the stopping offer is accepted. Using the last-steps look-ahead stopping rule (defined herein), the optimal stopping rule and the maximum expected reward are derived. It is shown that this stopping rule, that is occasionally effective to solve the multiple stopping type problem such as our problem having a specified stopping reward, is characterized by the discrete infinitesimal generator and Dynkin formula.

As example, we apply this to American call and put options on a geometric random walk. When only one stopping offer is allowed, for American option having uncertain exercise (declining) probability q, the followings are shown; (i) the optimal stopping rule is as same form as the one of the standard American option without uncertain exercise, (ii) the sequence of the optimal stopping boundary has the same values of the boundary of the standard American option, and (iii) the arbitrage-free price decrease to the value of p multiplying the price of the standard American option. The above (i) and (ii) implies that the declining probability q has no influence on the optimal exercise rule of the American option without declining probability. Further we show that (iv) the optimal stopping rule for the American option with uncertain stopping and m stopping offers has m stopping boundaries.

For both the path-dependent Russian and Asian call and put options with floating strike, it is shown that (i) and (ii) hold for these options and (v) the optimal stopping rule with m stopping offers is as same form as the one of American option.

Keywords discrete Dynkin formula; last-steps look-ahead stopping rule; uncertain stopping

References

1. K. Ano, S. Seko and A. Kurushima (2008). Optimal multiple stopping problem and its application to path-dependent options - discrete time case -. Bachelier Finance Society 5th World Congress 2008, London.
2. Y. S. Chow, H. Robbins and D. O. Siegmund, (1971). *Theory of Optimal Stopping.* Houghton Mifflin, Boston.
3. A. N. Shiryaev, (1999). *Essentials of Stochastic Finance.* World Scientific, River Edge.