proach (Abstract for the MTNS-2002) by R.F. Curtain,[*] H. Logemann[†] and O. Staffans[‡]

Absolute stability problems and their relations to positive-real conditions have played a prominent role in systems and control theory and have led to a number of important stability criteria for unity feedback controls applied to linear dynamical systems subject to static input or output nonlinearities.

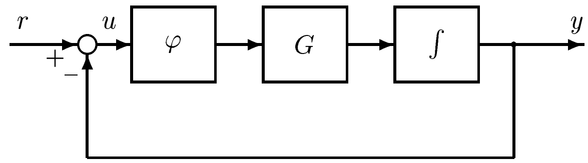

Figure 1

In this paper we study an absolute stability problem for the feedback system shown in Figure 1 and described by the integral equation

$$u(t) = r(t) - \int_0^t (G(\varphi \circ u))(\tau) d\tau \,.$$

The input-output operator G is linear, shift-invariant and bounded from $L^2(\mathbb{R}_+, U)$ into itself and $\varphi : U \to U$ is a locally Lipschitz nonlinearity, where U is a real separable Hilbert space. It is well-known that G can be represented by a transfer function $\mathbf{G}$ which is analytic and bounded on the open right-half of the complex plane. For simplicity we assume in this abstract that $\mathbf{G}$ admits an analytic extension to an open neighbourhood of 0 (this assumption will be weakened in the full paper). The main result of the paper shows in particular that if $\mathbf{G}(0)$ is invertible and if there exist a linear bounded self-adjoint operator $P : U \to U$, a linear, bounded, invertible operator $Q : U \to U$ with $Q\mathbf{G}(0) = [Q\mathbf{G}(0)]^* \geq 0$ and numbers $q \geq 0$ and $\varepsilon > 0$ such that

$$P + \frac{1}{2}\left(q\mathbf{G}(i\omega) + \frac{1}{i\omega} Q\mathbf{G}(i\omega) + q\mathbf{G}^*(i\omega) - \frac{1}{i\omega}\mathbf{G}^*(i\omega)Q^* \right) \geq \varepsilon I, \quad \text{a.a. } \omega \in \mathbb{R}\,,$$

then for any $r \in L^2(\mathbb{R}_+, U) + U$ with $\dot{r} \in L^2(\mathbb{R}_+, U)$ and for any locally Lipschitz gradient field $\varphi : U \to U$ with a non-negative potential and such that

$$\langle \varphi(v), Qv \rangle \geq \langle \varphi(v), P\varphi(v) \rangle, \quad \forall\, v \in U\,,$$

the solution u of the feedback system shown in Figure 1 exists on $\mathbb{R}_+$ (no finite escape-time), $u, y \in L^\infty(\mathbb{R}_+, U)$, $\varphi \circ u \in L^2(\mathbb{R}_+, U)$, $\lim_{t\to\infty} \varphi(u(t)) = 0$ and, under certain extra assumptions, $u(t)$ and $y(t)$ converge as $t \to \infty$. We emphasize that in contrast to previous results in the literature, our results consider feedback systems, where the linear part contains an integrator (meaning in particular that the linear system is not input-putput stable) and where at the same time the lower gain of the the nonlinearity φ is allowed to be equal to zero (which for example is the case for bounded nonlinearities such as saturation). One of the motivations for studying this situation is its importance in low-gain integral control in the presence of input nonlinearities, see Figure 2, where $\rho \in \mathbb{R}$ is a constant, $k \in \mathbb{R}$ is a gain parameter, $g : \mathbb{R}_+ \to \mathbb{R}$ is an output disturbance signal and G, $\mathbf{G}$ and φ are as before, but we assume a single-inpupt single-output context (i.e., $\dim U = 1$). The objective is to find a constant $k^* \in (0, \infty]$ in terms of the "system data" G and φ such that for all $k \in (0, k^*)$ the tracking error $e(t)$ becomes small in some sense as $t \to \infty$. We show that if $\mathbf{G}(0) > 0$, $g \in L^2(\mathbb{R}_+, \mathbb{R})$ with $t \mapsto \int_0^t g(\tau)d\tau \in L^2(\mathbb{R}_+, \mathbb{R}) + \mathbb{R}$, φ is non-decreasing and globally Lipschitz with Lipschitz constant $\lambda > 0$ and $\rho/\mathbf{G}(0) \in \operatorname{im}\varphi$, then the limit $\lim_{t\to\infty} u(t) =: u^\infty$ exists and $\varphi \circ u - \varphi(u^\infty)$, $e \in L^2(\mathbb{R}_+, \mathbb{R})$, provided that $k \in (0, 1/|\lambda f(G)|)$, where $f(G) := \sup_{q\geq 0}\{\text{ess inf}_{\omega\in\mathbb{R}} \operatorname{Re}\left[(q + 1/i\omega)\mathbf{G}(i\omega)\right]\}$. Under mild extra assumptions, the tracking error $e(t)$ converges to 0 as $t \to \infty$. Finally, we consider applications of the input-output results to the class of well-posed state-space systems.

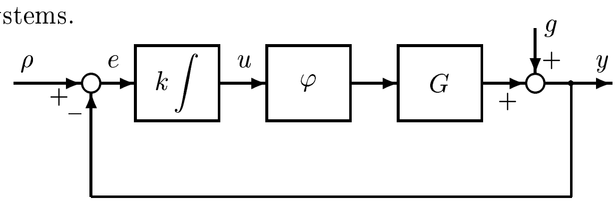

Figure 2

[*]Mathematics Institute, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands, email: R.F.Curtain@math.rug.nl

[†]Dept. of Mathematical Sciences, University of Bath, Claverton Down, Bath BA2 7AY, United Kingdom, email: hl@maths.bath.ac.uk

[‡]Dept. of Mathematics, Åbo Akademi University, Fänriksgatan 3B, FIN-20500 Åbo, Finland, email: olof.staffans@abo.fi