Passive and conservative state/signal systems in continuous time

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In this talk we discuss passive and conservative state/signal systems in continuous time. Such a system can be used to model, e.g., a passive linear electrical circuit containing lumped and/or distributed resistances, capacitors, inductors, and wave guides, etc. Most of the standard partial differential equations appearing in physics on can be written in state/signal form.

A passive state/signal system consists of three components: A) an internal Hilbert *state space* $\mathcal{X}$, B) a Kreĭn *signal space* $\mathcal{W}$ through which the system interacts with the external world, and C) a *generating subspace* V of the product space $\mathcal{X} \times \mathcal{X} \times \mathcal{W}$. The generating subspace is required to be maximally nonnegative with respect to a certain "energy" inner product and to satisfy an extra nondegeneracy condition. We dentote this system by $\Sigma = (V; \mathcal{X}, \mathcal{W})$. The set of all *classical trajectories* of Σ on some interval I consists of a continuously differentiable $\mathcal{X}$-valued state component x and a continuous $\mathcal{W}$-valued signal component w satisfying

$$(\dot{x}(t), x(t), w(t)) \in V, \qquad t \in I.$$

The set of all generalized trajectories of Σ is obtained from the family of all classical trajectories by a standard approxiation procedure.

By the *future behavior* of Σ we mean the set of all signal parts w of all stable trajectores (x, w) of Σ on $[0, \infty)$ satisfying the extra condition $x(0) = 0$. This set is a right-shift invariant subspace of $L^2([0, \infty); \mathcal{W})$ and it is maximal nonnegative with respect to the Kreĭn space inner product in $L^2([0, \infty); \mathcal{W})$ inherited from $\mathcal{W}$. Such a subspace is called a *passive future behavior*. Each passive future behavior can be realized as the future behavior of a passive state/signal system Σ, and it is possible to require Σ to have, for example, one of the following three sets of properties: a) Σ is observable and co-energy preserving; b) Σ is controllable and energy preserving; c) Σ is simple and conservative. Realizations within one of these classes are uniquel determined by the given future behavior. Furthermore, it is possible to construct *canonical* realizations, i.e., realizations with satisfy a), b), or c), and which are *uniquely determined by the given data*.

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