The Infinite-Dimensional Continuous Time Kalman–Yakubovich–Popov Inequality

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Introduction

Finite-Dimensional System

Linear finite-dimensional continuous-time-invariant systems are typically modeled by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad t \geq 0, \qquad x(0) = x_0, \\ y(t) &= Cx(t) + Du(t), \qquad t \geq 0. \end{aligned} \tag{1}$$

Here A, B, C, D, are operators (bounded for the moment),
$u(t) \in \mathcal{U}$ = the input space,
$x(t) \in \mathcal{X}$ = the state space,
$y(t) \in \mathcal{Y}$ = the output space (all Hilbert spaces).

A is the main operator,
B is the control operator,
C is the observation operator,
D is the feedthrough operator.

By a trajectory of this system we mean a triple of functions (x, u, y) satisfying (1) .

Scattering H-Passive System

The system (1) is scattering H-passive (or simply scattering passive if $H = 1_{\mathcal{X}}$) if all trajectories satisfy the condition

$$\frac{\mathrm{d}}{\mathrm{d}t} E_H(x(t)) \leq j(u(t), y(t)) \text{ a.e. on } (s, \infty), \tag{2}$$

where E_H is a positive storage function (Lyapunov function)

$$E_H(x) = \langle Hx, x \rangle_{\mathcal{X}}, \quad H > 0,$$

and j is the quadratic scattering supply rate

$$j(u, y) = \|u\|_{\mathcal{U}}^2 - \|y\|_{\mathcal{Y}}^2 = \left\langle \left[\begin{smallmatrix} u \\ y \end{smallmatrix}\right], \begin{bmatrix} 1_{\mathcal{U}} & 0 \\ 0 & -1_{\mathcal{Y}} \end{bmatrix} \left[\begin{smallmatrix} u \\ y \end{smallmatrix}\right] \right\rangle_{\mathcal{U} \oplus \mathcal{Y}} .$$

The Kalman–Yakubovich–Popov Inequality

Condition (2) is equivalent to

$$2\Re\langle Ax + Bu, Hx\rangle + \|Cx + Du\|^2 \leq \|u\|^2, \qquad x \in \mathcal{X}, u \in \mathcal{U}, \tag{3}$$

which is usually rewritten in the form

$$\begin{bmatrix} HA + A^*H + C^*C & HB + C^*D \\ B^*H + D^*C & D^*D - 1_{\mathcal{U}} \end{bmatrix} \leq 0. \tag{4}$$

This inequality is named after Kalman [Kal63], Yakubovich [Yak62], and Popov [Pop61] (here in continuous time with scattering supply rate).

Problem: Find conditions on the coefficients A, B, C, D under which the KYP inequality has at least one solution $H > 0$.

The Transfer Function and the Schur Class

We define the transfer function of the system (1) by

$$\widehat{\mathfrak{D}}(z) = D + C(z-A)^{-1}B, \qquad z \in \rho(A).$$

We also introduce the Schur class $S(\mathcal{U},\mathcal{Y};\mathbb{C}^+)$ of holomorphic contractive functions $\widehat{\mathfrak{D}}$ defined on $\mathbb{C}^+$ with values in $\mathcal{B}(U,Y)$.

Here $\mathbb{C}^+ = \{z \in \mathbb{C} \mid \Re z > 0\}$.

Controllability and Observability

A finite-dimensional system is minimal if the dimension of the state space is the smallest one among all systems with the same transfer function $\mathfrak{D}$.

The (finite-dimensional) system (1) is controllable if, given any $z_0 \in \mathcal{X}$ and $T > 0$, there exists some continuous function u on $[0, T]$ such that the solution of (1) with $x(0) = 0$ satisfies $x(T) = z_0$.

The system (1) is observable if it has the following property: if both the input function u and the output function y vanish on some interval $[0, T]$ with $T > 0$, then necessarily the initial state x_0 is zero.

Theorem 1 (Kalman). *A finite-dimensional system is minimal if and only if it is controllable and observable* **.**

The Finite-Dimensional KYP Theorem

Theorem 2 (Kalman–Yakubovich–Popov). *Let* $\Sigma = \left(\left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]; \mathcal{X}, \mathcal{U}, \mathcal{Y}\right)$ *be a system with* $\dim \mathcal{X} < \infty$ *and transfer function* $\widehat{\mathfrak{D}}$*.*

(i) *If the KYP inequality* (4) *has a solution* $H > 0$ *then* $\mathbb{C}^+ \subset \rho(A)$ *and* $\widehat{\mathfrak{D}}|_{\mathbb{C}^+} \in S(\mathcal{U}, \mathcal{Y}; \mathbb{C}^+)$*. (Here* $\widehat{\mathfrak{D}}|_{\mathbb{C}^+}$ *is the restriction of* $\widehat{\mathfrak{D}}$ *to* $\mathbb{C}^+$*.)*

(ii) *If* Σ *is minimal and* $\widehat{\mathfrak{D}}|_{\mathbb{C}^+} \in S(\mathcal{U}, \mathcal{Y}; \mathbb{C}^+)$*, then the KYP inequality* (4) *has a solution* H*, i.e.,* Σ *is scattering* H*-passive for some* $H > 0$ **.**

Infinite-Dimensional Setting

Infinite-Dimensional Extensions

In the seventies the classical results on the KYP inequalities were extended to systems with $\dim \mathcal{X} = \infty$ by Yakubovich and his students and collaborators (see [Yak74, Yak75, LY76] and the references listed there). There is now also a rich literature on this subject; see, e.g., the discussion in [Pan99] and the references cited there.

However, as far as we know, in these and all later (continuous time) generalizations it was assumed that either H itself is bounded or H^{-1} is bounded.

The infinite-dimensional discrete time KYP inequality with scattering supply rate was studied by Arov, Kaashoek and Pik in [AKP05]. There both H and H^{-1} were allowed to be unbounded.

Here we extend their result to continuous time .

Why Unbounded H and H^{-1}?

The operator H is very sensitive to the choice of the original norm in the state space, and the boundedness of H and H^{-1} depends entirely on the choice of the original norm in $\mathcal{X}$.

By allowing both H and H^{-1} to be unbounded we can use an analogue of the standard finite-dimensional procedure to determine whether a given transfer function θ is a Schur function or not, namely to choose an arbitrary minimal realization of θ, and then check whether the KYP inequality (4) has a positive (generalized) solution. This procedure does not work if we insist on having H or H^{-1} bounded.

Thus, by allowing H and H^{-1} to be unbounded we enlarge the class of realizations that can be used, and thereby simplify the modeling process .

The Main Operator A

We shall use the continuous time setting of, e.g., [AN96], [Šmu86], [Sal89], [Sta05].

Postulate I. The main operator A is the generator of a C_0-semigroup $\mathfrak{A}^t$, $t \geq 0$, on the Hilbert space $\mathcal{X}$.

Denote

$$\mathcal{D}(A) =: \mathcal{X}_1 \subset \mathcal{X} \subset \mathcal{X}_{-1} := [\mathcal{D}(A^*)]^*.$$

The main operator A has a unique extension to a bounded linear operator $\mathcal{X} \to \mathcal{X}_{-1}$, which we denote by $\widehat{A}$.

The Control Operator B

Postulate II. The control operator B satisfies $B \in \mathcal{B}(\mathcal{U}, \mathcal{X}_{-1})$.

The first equation in (1) will be interpreted to take its values in $\mathcal{X}_{-1}$):

$$\dot{x}(t) = \widehat{A}x(t) + Bu(t), \qquad t \geq 0, \qquad x(0) = x_0. \tag{5}$$

This equation has the generalized solution ($\forall u \in C(\mathbb{R}^+; \mathcal{U})$)

$$x(t) = \widehat{\mathfrak{A}}^t x_0 + \int_0^t \widehat{\mathfrak{A}}^{t-s} Bu(s)ds \quad . \tag{6}$$

The Main/Control Operator $A\&B$

We define the combined main/control operator $A\&B$ by $A\&B = \begin{bmatrix} \widehat{A} & B \end{bmatrix}|_{\mathcal{D}(A\&B)}$, where

$$\left[\begin{smallmatrix} x \\ u \end{smallmatrix} \right] \in \mathcal{D}(A\&B) \Leftrightarrow \widehat{A}x + Bu \in \mathcal{X}.$$

If we choose smoother data, for example

$$u \in W^2_{2,\mathrm{loc}}(\mathbb{R}^+;\mathcal{U}) \text{ and } \left[\begin{smallmatrix} x_0 \\ u(0) \end{smallmatrix} \right] \in \mathcal{D}(A\&B), \tag{7}$$

then x is continuously differentiable in $\mathcal{X}$, $\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \in \mathcal{D}(A\&B)$ for all $t \in \mathbb{R}^+$, and (5) becomes

$$\dot{x}(t) = A\&B \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \qquad t \geq 0, \qquad x(0) = x_0 \quad . \tag{8}$$

The Observation/Feedthrough Operator $C\&D$

We fix some operator $C\&D\colon \left[\begin{smallmatrix}\mathcal{X}\\ \mathcal{U}\end{smallmatrix}\right] \to \mathcal{Y}$, and define the output by

$$y(t) = C\&D \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}.$$

Postulate III. $C\&D \in \mathcal{B}(\mathcal{D}(A\&B), \mathcal{Y})$ (boundedness with respect to the graph norm of $A\&B$).

We can recover the observation operator C from $C\&D$: We have $\left[\begin{smallmatrix}x\\ 0\end{smallmatrix}\right] \in \mathcal{D}(A\&B)$ if and only if $x \in \mathcal{X}_1$, so we can define $C \in \mathcal{B}(\mathcal{X}_1, \mathcal{Y})$ by

$$Cx = C\&D \left[\begin{smallmatrix}x\\ 0\end{smallmatrix}\right], \qquad x \in \mathcal{X}_1.$$

The system need not have a feed-through operator .

The Transfer Function $\widehat{\mathfrak{D}}$ and the System Node

It can be proved that

$$\begin{bmatrix}(z-\widehat{A})^{-1}Bu\\ u\end{bmatrix}\in\mathcal{D}(A\&B)$$

for all $z\in\rho(A)$ and $u\in\mathcal{U}$. We can therefore define the transfer function $\widehat{\mathfrak{D}}$ of the system (9) by

$$\widehat{\mathfrak{D}}(z)=C\&D\left[\begin{smallmatrix}(z-\widehat{A})^{-1}B\\ 1_{\mathcal{U}}\end{smallmatrix}\right],\quad z\in\rho(A).$$

When Postulates I–III hold we call

$$\Sigma:=\left(\left[\begin{smallmatrix}A\&B\\ C\&D\end{smallmatrix}\right];\mathcal{X},\mathcal{U},\mathcal{Y}\right)$$

a system node. Here $S:=\left[\begin{smallmatrix}A\&B\\ C\&D\end{smallmatrix}\right]$ is the system operator, with $\mathcal{D}(S)=\mathcal{D}(A\&B)$.

The Dynamics of the System Node

Thus, the extension of (1) that we shall use here is

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = S \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A\&B \\ C\&D \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \quad t \geq 0. \tag{9}$$

This equation has a unique solution $x \in C^1(\mathbb{R}^+, \mathcal{X})$ (given by (6)) whenever $u \in W^2_{2,\mathrm{loc}}(\mathbb{R}^+;\mathcal{U})$ and $\left[\begin{smallmatrix} x_0 \\ u(0) \end{smallmatrix}\right] \in \mathcal{D}(A\&B)$, and the output y satisfies $y \in C(\mathbb{R}^+, \mathcal{Y})$.

System Node Summary: A generates a C_0 semigroup on $\mathcal{X}$ with generator A,
$B \in \mathcal{B}(\mathcal{U}, \mathcal{X}_{-1})$,
$\mathcal{D}(A\&B) = \left\{ \left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in \left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{U} \end{smallmatrix}\right] \,\middle|\, \widehat{A}x + Bu \in \mathcal{X} \right\}$,
$A\&B = \begin{bmatrix} \widehat{A} & B \end{bmatrix} |_{\mathcal{D}(A\&B)}$,
$C\&D \in \mathcal{B}(\mathcal{D}(A\&B), \mathcal{Y})$ (graph norm on $\mathcal{D}(A\&B)$) .

H-Passivity

H-Passivity: General Setup

We allow both the storage operator $H > 0$ and its inverse H^{-1} to be unbounded.

This means that one must be very careful about the domains on which the different operators act.

We rewrite the storage function E_H in the form

$$E_H(x) = \|\sqrt{H}x\|^2, \quad x \in \mathcal{D}(\sqrt{H}),$$

where $\sqrt{H} > 0$ is the self-adjoint square root of H. This is equivalent to replacing the operator $H > 0$ by the corresponding (closed) quadratic form induced by H .

H-Passive System Node $\Sigma := \left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]; \mathcal{X}, \mathcal{U}, \mathcal{Y}\right)$: Definition

(i) $H = H^* > 0$. Denote $Q := \sqrt{H}$.

(ii) (Invariance): If $u \in W^{2,2}_{\mathrm{loc}}([0,\infty);\mathcal{U})$ and $\left[\begin{smallmatrix} x_0 \\ u(0) \end{smallmatrix}\right] \in \mathcal{D}(A\&B)$ with $x_0 \in \mathcal{D}(Q)$ and $A\&B \left[\begin{smallmatrix} x_0 \\ u(0) \end{smallmatrix}\right] \in \mathcal{D}(Q)$, then the solution x of (9) satisfies $x(t)$, $\dot{x}(t) \in \mathcal{D}(Q)$ for all $t \geq 0$, and both Qx and $Q\dot{x}$ are continuous in $\mathcal{X}$ on $[0,\infty)$.

(iii) (Energy Inequality): Each solution of the type described in (ii) satisfies

$$\langle Qx(t), Qx(t)\rangle_{\mathcal{X}} + \int_0^t \|y(s)\|^2_{\mathcal{Y}}\, ds \leq \langle Qx(0), Qx(0)\rangle_{\mathcal{X}} + \int_0^t \|u(s)\|^2_{\mathcal{U}}\, ds. \quad (10)$$

The KYP Inequality: General Setup

We rewrite the KYP inequality in the same way ($H = H^* > 0$ and $Q = \sqrt{H}$):

$$2\Re\langle Q[A\&B]\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right], Qx\rangle + \|C\&D\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right]\|^2 \le \|u\|^2. \tag{11}$$

This inequality should hold for the following natural set of data:

$$x \in \mathcal{D}(Q), \quad \left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in \mathcal{D}(A\&B), \quad A\&B\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in \mathcal{D}(Q). \tag{12}$$

Under Postulate I, the resolvent set $\rho(A)$ of A contains the right half-plane $\mathbb{C}^+_\omega = \{z \in \mathbb{C} \mid \Re z > \omega\}$, where

$$\omega = \limsup_{t\to\infty} \frac{1}{t} \log\|\mathfrak{A}^t\|.$$

We let $\rho^+_\infty(A)$ be the (connected) component of $\rho(A) \cap \mathbb{C}^+$ which contains $\mathbb{C}^+_\omega$.

The Generalized KYP Inequality for $\Sigma := \left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]; \mathcal{X}, \mathcal{U}, \mathcal{Y}\right)$

(i) $H = H^* > 0$. Denote $Q := \sqrt{H}$.

(ii) $(\lambda - A)^{-1}\mathcal{D}(Q) \subset \mathcal{D}(Q)$ for some $\lambda \in \rho_\infty^+(A)$.

(iii) $(\lambda - \widehat{A})^{-1}B\mathcal{U} \subset \mathcal{D}(Q)$ for some $\lambda \in \rho_\infty^+(A)$.

(iv) The operator QAQ^{-1} is closable.

(v) For all $\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right] \in \mathcal{D}(A\&B)$ with $x_0 \in \mathcal{D}(Q)$ and $A\&B\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right] \in \mathcal{D}(Q)$ we have

$$2\Re\langle Q[A\&B]\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right], Qx_0\rangle_{\mathcal{X}} + \|C\&D\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right]\|_{\mathcal{Y}}^2 \le \|u_0\|_{\mathcal{U}}^2 \quad .$$

H-Passivity $\Leftrightarrow$ KYP-Inequality

Theorem 3. *Let $\Sigma := (S; \mathcal{X}, \mathcal{U}, \mathcal{Y})$ be a system node, and let $H = H^* > 0$. Then the following two conditions are equivalent:*

(i) Σ *is* H*-passive.*

(ii) H *is a generalized solution of the KYP-inequality.*

One direction of the proof is fairly simple (the one which says that H-passivity of Σ implies that H is a solution of the generalized KYP-inequality).

The proof of the converse is more difficult, especially the proof of the invariance condition .

The Modified Generalized KYP Inequality for Σ

(i) $H = H^* > 0$. Denote $Q := \sqrt{H}$.

(ii') $\mathfrak{A}^t \mathcal{D}(Q) \subset \mathcal{D}(Q)$ for all $t \in \mathbb{R}^+$, and the function $t \mapsto Q\mathfrak{A}^t x_0$ is continuous on $\mathbb{R}^+$ (with values in $\mathcal{X}$) for all $x_0 \in \mathcal{D}(Q)$.

(iii') $\mathcal{R}(\mathfrak{B}) \subset \mathcal{D}(Q)$, where $\mathfrak{B}$ is the input map of Σ (the map from the control to the state).

(v) For all $\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right] \in \mathcal{D}(A\&B)$ with $x_0 \in \mathcal{D}(Q)$ and $A\&B \left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right] \in \mathcal{D}(Q)$ we have

$$2\Re\langle Q[A\&B]\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right], Qx_0\rangle_{\mathcal{X}} + \|C\&D\left[\begin{smallmatrix} x_0 \\ u_0 \end{smallmatrix}\right]\|_{\mathcal{Y}}^2 \le \|u_0\|_{\mathcal{U}}^2.$$

This is an exact analogue of the discrete time case .

Controllability, Observability, Restricted Schur Class

Σ is controllable if $X_\Sigma^C = \mathcal{X}$, where $X_\Sigma^C = \bigvee_{\lambda \in \rho_\infty^+(A)} \mathcal{R}((\lambda - A)^{-1}B)$.

Σ is observable if $X_\Sigma^U = 0$, where $X_\Sigma^U = \bigcap_{\lambda \in \rho_\infty^+(A)} \mathcal{N}(C(\lambda - A)^{-1})$.

Σ is minimal if Σ is both controllable and observable.

Let Ω be an open connected subset of $\mathbb{C}^+$. A function θ belongs to the restricted Schur class $S(\mathcal{U}, \mathcal{Y}; \Omega)$ if it is the restriction to Ω of a function in $S(\mathcal{U}, \mathcal{Y}, \mathbb{C}^+)$.

Connection H-Passivity $\leftrightarrow$ Transfer Function

Theorem 4. *Let $\widehat{\mathfrak{D}}$ be the transfer function of $\Sigma := \left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]; \mathcal{X}, \mathcal{U}, \mathcal{Y}\right)$.*

(i) *If the KYP inequality* (11) *has a solution $H = H^* > 0$, then $\widehat{\mathfrak{D}}|_{\rho_\infty^+(A)} \in S(\mathcal{U}, \mathcal{Y}; \rho_\infty^+(A))$.*

(ii) *If Σ is minimal and $\widehat{\mathfrak{D}}|_{\rho_\infty^+(A)} \in S(\mathcal{U}, \mathcal{Y}; \rho_\infty^+(A))$, then the KYP inequality* (11) *has a solution $H = H^* > 0$ such that $\Sigma_H = \Sigma$ with the norm $\|x\|_{\mathcal{X}_H} = \|\sqrt{H}x\|_{\mathcal{X}}$ is minimal.*

Note: The KYP inequality says that Σ_H is scattering passive (with $H = 1_{\mathcal{X}_H}$) .

Ordering of Solutions of KYP Inequality

We denote the set of all solution $H = H^* > 0$ satisfying the additional minimality condition in poart (ii) above by $\mathcal{L}_\Sigma^{\min}$.

Theorem 5. *Let Σ be a minimal continuous time-invariant system of the type* (9) *that satisfies Postulates I–III and the additional condition $\widehat{\mathfrak{D}}|_{\rho_\infty^+(A)} \in S(\mathcal{U}, \mathcal{Y}; \rho_\infty^+(A))$. Then $\mathcal{L}_\Sigma^{\min} \neq \emptyset$, and it contains a minimal element $H_\circ$ and a maximal element $H_\bullet$:*

$$H_\circ \preceq H \preceq H_\bullet \quad \forall H \in \mathcal{L}_\Sigma^{\min}.$$

$H_1 \preceq H_2 \Leftrightarrow \mathcal{D}(\sqrt{H_2}) \subset \mathcal{D}(\sqrt{H_1})$ and $\|\sqrt{H_1}x\| \leq \|\sqrt{H_2}x\|\ \forall x \in \mathcal{D}(\sqrt{H_2})$.

$E_{H_\circ}(\cdot)$ is the available storage, and $E_{H_\bullet}(\cdot)$ is the required supply (Willems).

$H_\circ$ is the optimal and $H_\bullet$ is the $*$-optimal solution of the KYP inequality (Arov) .

H-stability

Let $H = H^* > 0$. We define the notion of H-stability as stability with respect to the norm

$$\|x\|_H = \|\sqrt{H}x\|_{\mathcal{X}}$$

for the original system (1), and with respect to the norm

$$\|x\|_{H^{-1}} = \|\sqrt{H^{-1}}x\|_{\mathcal{X}}$$

for the adjoint system. In the finite-dimensional case this is not important: all norms in $\mathcal{X}$ are equivalent. However,

In the infinite-dimensional case the H-stability of the system depends strongly on H (where H is a solution of the generalized KYP inequality).

For example, it may be exponentially H-stable for some H, and not even strongly H-stable for some other H .

An Example

Take an exponentially damped heat equation with damping coefficient $\alpha \geq 1$ on a semi-infinite bar with Neumann control and Dirichlet observation.

The transfer function of this system is the Schur function

$$\widehat{\mathfrak{D}}(z) = \frac{1}{\sqrt{z+\alpha}}, \qquad z \in \mathbb{C}^+.$$

The standard heat equation realization is self-adjoint and exponentially stable with decay rate $-\alpha$. This realization is a balanced passive realization in the sense of [Sta05].

However, the system is not even strongly $H_\bullet$-stable (where $H_\bullet$ is the maximal solution of the generalized KYP inequality, corresponding to the required supply). It is not adjoint strongly $H_\circ$-stable either (where $H_\circ$ is the minimal solution of the generalized KYP inequality, corresponding to the available storage) .

Further Extensions

Similar results are true in the impedance and transmission settings, as can be shown by using the technique developed in [AS05a, AS05b, AS05c].

Instead of working with energy inequalities we can also work with energy balance equations. In this case the system will be forward conservative or even conservative.

Analogous results also hold for the quadratic cost minimization problem and its dual. The advantage with this approach is that we get rid of the finite cost condition. This is current joint work with Mark Opmeer .

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