Passive and Conservative Infinite-Dimensional Impedance and Scattering Systems (from a Personal Point of View)

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Let U be a Hilbert space. By a $\mathcal{L}(U)$*-valued positive analytic function* on the open right half-plane we mean an analytic function which satisfies the condition $\widehat{\mathfrak{D}} + \widehat{\mathfrak{D}}^* \geq 0$. This function need not be *proper*, i.e., it need not be bounded on any right half-plane. We give a complete answer to the question under what conditions such a function can be realized as the transfer function of a *impedance passive system*. By this we mean a continuous time state space system whose control and observation operators are not more unbounded than the (main) semigroup generator of the system, and in addition, there is a certain energy inequality relating the absorbed energy and the internal energy. The system is (impedance) *energy preserving* if this energy inequality is an equality, and it is *conservative* if both the system and its dual are energy preserving. A typical example of an impedance conservative system is a system of hyperbolic type with collocated sensors and actuators. We prove that a passive realization exists if and only if a conservative realization exists, and that this is true if and only if $\lim_{s\to+\infty} \frac{1}{s}\widehat{\mathfrak{D}}(s)u = 0$ for every $u \in U$. The physical interpretation of this condition is that the input-output response is not allowed to contain a pure derivative action. We furthermore show that the so called *diagonal transform* (which is a particular rescaled feedback/feedforward transform) maps an *impedance* passive (or energy preserving or conservative) system into a (well-posed) *scattering* passive (or energy preserving or conservative) system. This implies that if we apply negative output feedback to a impedance passive system, then the resulting system is both well-posed and energy stable. Finally, we study *lossless* scattering systems, i.e., scattering conservative systems whose transfer functions are inner.

Keywords: Dissipative, energy preserving, lossless, proper, collocated sensors and actuators, positive real, Caratheodory-Nevanlinna function, Titchmarsh-Weyl function, bounded real lemma, Kalman-Yakubovich-Popov lemma, feedback, Cayley transform, diagonal transform.