Well-Posed Linear Systems, Lax–Phillips Scattering, and L^p-Multipliers

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We discuss the connection between Lax–Phillips scattering theory and the theory of well-posed linear systems, and show that the latter theory is a natural extension of the former. As a consequence of this, there is a close connection between the Lax–Phillips generator and the generators of the corresponding well-posed linear system. All the essential information about these two systems is contained in the system operator $S_\Sigma = \left[\begin{smallmatrix} A & B \\ C\&D \end{smallmatrix}\right]$, where A is the generator of the (central) semigroup, B is the control operator, and $C\&D$ is the combined observation/feedthrough operator. In the important Hilbert space case this system operator can be written in the more familiar form $S_\Sigma = \left[\begin{smallmatrix} A & B \\ \overline{C} & D \end{smallmatrix}\right]$, where $\overline{C}$ is a (not necessarily uniquely determined) observation operator and D is the corresponding (generalized) feedthrough operator. The system operator is closed and densely defined. In the reflexive case the adjoint of S_Σ is the system operator of the dual system. We give formulas for the Lax–Phillips generator and resolvent in terms of the system operator. By applying the Hille–Yoshida theorem to the Lax–Phillips semigroup we get necessary and sufficient conditions for the L^p-admissibility or joint L^p-admissibility of a control operator B and an observation operator C. This leads to a criterion for an H^∞-function to be an L^p-multiplier.