Spectral Decomposition and Invariant Manifolds for some Functional Partial Differential Equations

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$$\frac{d}{dt}(x+\mu*x)-Ax-\nu*x=f \text{ on } [0,+\infty),$$
$$x=f \text{ on } (-\infty,0],$$

as well as a nonlinear perturbation of the corresponding homogeneous equation. Here A is the generator of an analytic semigroup on a Hilbert space H, and μ and ν are operator-valued dominated measures with values in $L(H)$ and $L(D(A),H)$ respectively. Under the assumption that the operator given by the Laplace transform of the left-hand side of the equation is boundedly invertible on some right half-plane and on a line in the left half-plane, parallel to the imaginary axis, we decompose the solutions into components with different exponential growth rates. We construct projectors onto the stable and unstable subspaces, which are then used for the construction of stable and unstable manifolds for the nonlinear equation, which can have a fully nonlinear character. The results are applied to two equations of parabolic type. Moreover, the spectrum of the generator of the translation semigroup in various weighted spaces is determined, including the stable and unstable subspaces of our problem.