Bi-Inner Dilations and Bi-Stable Passive Scattering Realizations of Schur Class Operator-Valued Functions

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Let $S(U;Y)$ be the class of all Schur functions (analytic contractive functions) whose values are bounded linear operators mapping one separable Hilbert space U into another separable Hilbert space Y, and which are defined on a domain $\Omega \subset \mathbb{C}$, which is either the open unit disk $\mathbb{D}$ or the open right half-plane $\mathbb{C}^+$. In the development of the Darlington method for passive linear time-invariant input/state/output systems (by Arov, Dewilde, Douglas and Helton) the following question arose: do there exist simple necessary and sufficient conditions under which a function $\theta \in S(U;Y)$ has a bi-inner dilation $\Theta = \begin{bmatrix} \theta_{11} & \theta \\ \theta_{21} & \theta_{22} \end{bmatrix}$ mapping $U_1 \oplus U$ into $Y \oplus Y_1$; here U_1 and Y_1 are two more separable Hilbert spaces, and the requirement that Θ is bi-inner means that Θ is analytic and contractive on Ω and has unitary nontangential limits a.e. on $\partial\Omega$. There is an obvious well-known necessary condition: there must exist two functions $\psi_r \in S(U;Y_1)$ and $\psi_l \in S(U_1;Y)$ (namely $\psi_r = \theta_{22}$ and $\psi_l = \theta_{11}$) satisfying $\psi_r^*(z)\psi_r(z) = I - \theta^*(z)\theta(z)$ and $\psi_l(z)\psi_l^*(z) = I - \theta(z)\theta^*(z)$ for almost all $z \in \partial\Omega$. We prove that this necessary condition is also sufficient. Our proof is based on the following facts. 1) A solution ψ_r of the first factorization problem mentioned above exists if and only if the minimal optimal passive realization of θ is strongly stable. 2) A solution ψ_l of the second factorization problem exists if and only if the minimal $*$-optimal passive realization of θ is strongly $*$-stable (the adjoint is strongly stable). 3) The full problem has a solution if and only if the balanced minimal passive realization of θ is strongly bi-stable (both strongly stable and strongly $*$-stable). This result seems to be new even in the case where θ is scalar-valued.

Keywords: Darlington method, optimal passive realization, $*$-optimal passive realization, balanced passive realization.