Summary of Ph.D. thesis: **Towards input/output-free modelling of linear infinite-dimensional systems in continuous time**

Mikael Kurula

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The 230-page thesis consists of the following three articles: [KS10b], [Kur10a] and [KZvdSB10] and a 70-page summary tying them together. The summary can be found at `http://urn.fi/URN:ISBN:978-952-12-2418-8`. The full thesis is available as a hard copy with ISBN 978-952-12-2410-2, and it can be obtained from me on request by sending an e-mail to *mkurula@abo.fi*.

In the thesis I extend parts of a new approach to infinite-dimensional systems theory known as *state/signal systems theory* (see [Sta06] and its references for an overview) from discrete-time systems to continuous-time systems. In the state/signal approach there is a minimal distinction between inputs and outputs, and this is an advantage, e.g., when one considers interconnections of systems and properties inherent in a system. Controllability, observability, and passivity are examples of properties which can be defined without distinguishing inputs from outputs.

All systems considered in the thesis are linear, and in general they are infinite-dimensional. Therefore unbounded linear operator theory provides the main tools for the study, and most of the substance of the thesis lies in the technical details of the appended articles. The main emphasis is on well-posed systems [KS10b], in particular passive and conservative systems [Kur10a], and their interconnection theory [KZvdSB10].

I now describe the contents of the thesis in more detail.

1 Introducing state/signal systems

First consider an input/state/output (i/s/o) system

$$\begin{bmatrix}\dot x(t)\\ y(t)\end{bmatrix} = \begin{bmatrix}A\&B\\ C\&D\end{bmatrix}\begin{bmatrix}x(t)\\ u(t)\end{bmatrix}, \quad t \geq 0,\ x(0) = x_0, \tag{1.1}$$

with an unbounded *system node* $\left[\begin{smallmatrix}A\&B\\ C\&D\end{smallmatrix}\right]$ with state space $\mathcal{X}$, input space $\mathcal{U}$, and output space $\mathcal{Y}$, all Hilbert spaces. See [Kur10b, Def. 2.4] for a definition of the concept i/s/o system node.

In order to turn the system (1.1) into a state/signal (s/s) system, we first combine the input u and output y into a single external signal $w := \left[\begin{smallmatrix}y\\ u\end{smallmatrix}\right]$. Then we replace (1.1) by the condition

$$\begin{gathered}\begin{bmatrix}\dot x(t)\\ x(t)\\ w(t)\end{bmatrix} \in V, \quad t \geq 0,\ x(0) = x_0, \quad \text{where}\\ V = \begin{bmatrix}A\&B\\ \begin{bmatrix}1 & 0\end{bmatrix}\\ \left[\begin{smallmatrix}C\&D\\ [0\ 1]\end{smallmatrix}\right]\end{bmatrix} \operatorname{dom}\left(\left[\begin{smallmatrix}A\&B\\ C\&D\end{smallmatrix}\right]\right).\end{gathered} \tag{1.2}$$

In this way we obtain a system without a priori given inputs and outputs. The external signal w can then easily be decomposed into an input u and an output y in different ways, depending on what the situation demands. The system properties are described in terms of geometrical properties of V, rather than in terms of $\left[\begin{smallmatrix}A\&B\\ C\&D\end{smallmatrix}\right]$, which is merely one of many i/s/o representations of the system. We return to this point later.

We now give the general definition of a continuous-time s/s system without assuming an i/s/o representation of the type (1.2).

Definition 1.1. Let $\mathcal{X}$ be a Hilbert space and $\mathcal{W}$ a Kreĭn space, and let V be a closed subspace of $\left[\begin{smallmatrix}\mathcal{X}\\ \mathcal{X}\\ \mathcal{W}\end{smallmatrix}\right]$. The space $\mathfrak{V}$ of *classical trajectories* generated by V on $\mathbb{R}^+$ consists of all pairs $\left[\begin{smallmatrix}x\\ w\end{smallmatrix}\right] \in \left[\begin{smallmatrix}C^1(\mathbb{R}^+;\mathcal{X})\\ C(\mathbb{R}^+;\mathcal{W})\end{smallmatrix}\right]$, such that $\left[\begin{smallmatrix}\dot x(t)\\ x(t)\\ w(t)\end{smallmatrix}\right] \in V$ for all $t > 0$.

We say that $(V;\mathcal{X},\mathcal{W})$ is a *state/signal node* if:

(i) The *generating subspace* V has the property $\left[\begin{smallmatrix}z\\ 0\\ 0\end{smallmatrix}\right] \in V \Longrightarrow z = 0$.

(ii) Every vector in V can be chosen as initial data for a classical trajectory on $\mathbb{R}^+$:

$$\forall \begin{bmatrix}z_0\\ x_0\\ w_0\end{bmatrix} \in V\ \exists \begin{bmatrix}x\\ w\end{bmatrix} \in \mathfrak{V}: \quad \begin{bmatrix}\dot x(0)\\ x(0)\\ w(0)\end{bmatrix} = \begin{bmatrix}z_0\\ x_0\\ w_0\end{bmatrix}.$$

A pair $\left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right] \in \left[\begin{smallmatrix} C(\mathbb{R}^+;\mathcal{X}) \\ L^2_{loc}(\mathbb{R}^+;\mathcal{W}) \end{smallmatrix}\right]$ is a *generalised trajectory* generated by V on $\mathbb{R}^+$ if there exists a sequence of classical trajectories that converges to $\left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right]$ in $\left[\begin{smallmatrix} C(\mathbb{R}^+;\mathcal{X}) \\ L^2_{loc}(\mathbb{R}^+;\mathcal{W}) \end{smallmatrix}\right]$. We denote the space of generalised trajectories on $\mathbb{R}^+$ by $\mathfrak{W}$.

By the *state/signal system* induced by the s/s node $(V;\mathcal{X},\mathcal{W})$ we mean the triple $(\mathfrak{W};\mathcal{X},\mathcal{W})$. ⧫

Thus, a s/s system is essentially a subspace of $\left[\begin{smallmatrix} C(\mathbb{R}^+;\mathcal{X}) \\ L^2_{loc}(\mathbb{R}^+;\mathcal{W}) \end{smallmatrix}\right]$, whose elements are generalised trajectories induced by the generating subspace V of some s/s node $(V;\mathcal{X},\mathcal{W})$.

Remark 1.2. By definition a s/s node $(V;\mathcal{X},\mathcal{W})$ determines the s/s system $(\mathfrak{W};\mathcal{X},\mathcal{W})$ it induces uniquely, but at the time I wrote my thesis it was not clear if the converse is true. In [KS10a], however, we establish that a s/s system $(\mathfrak{W};\mathcal{X},\mathcal{W})$ determines its inducing s/s node uniquely by

$$V = \left\{ \left[\begin{smallmatrix} \dot{x}(0) \\ x(0) \\ w(0) \end{smallmatrix}\right] \;\middle|\; \begin{bmatrix} x \\ w \end{bmatrix} \in \mathfrak{W} \cap \begin{bmatrix} C^1(\mathbb{R}^+;\mathcal{X}) \\ C(\mathbb{R}^+;\mathcal{W}) \end{bmatrix} \right\}. \tag{1.3}$$

The equality (1.3) follows from the fact that every generalised trajectory, which has the required smoothness, is in fact a classical trajectory. This insight clarifies the continuous-time s/s theory significantly. In particular, the concept of a *maximal generating subspace*, which was discussed in Section 6 of [KS10b], becomes superfluous. ⧫

Every i/s/o system node $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$ induces a s/s node $\left(V;\mathcal{X},\left[\begin{smallmatrix} \mathcal{Y} \\ \mathcal{U} \end{smallmatrix}\right]\right)$ through (1.2), but the converse is not true.

Definition 1.3. Let $(V;\mathcal{X},\mathcal{W})$ be a s/s node.

If there exists a direct-sum decomposition $\mathcal{W} = \mathcal{U} \dotplus \mathcal{Y}$, such that V can be written on the form (1.2) for an i/s/o system node $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$, then $(\mathcal{U},\mathcal{Y})$ is said to be an *admissible input/output (i/o) pair* and $(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right];\mathcal{X},\mathcal{U},\mathcal{Y})$ is the corresponding *i/s/o representation.*

A s/s system $(V;\mathcal{X},\mathcal{W})$ is *well-posed* if it has at least one well-posed i/s/o representation, see [Kur10b, Def. 2.5] for a definition. ⧫

The intuitive interpretation of passivity of a system is a lack of internal energy sources, and a conservative system is a passive system with no internal energy sinks. In i/s/o control theory, however, there are several different kinds of passivity, such scattering and impedance, depending on how the energy exchange with the surrounding world is takes place. One of the main ideas of the s/s theory is to unify the theories of these types of passive i/s/o systems, so that one single proof can be given for the s/s system, rather than separate but similar proofs for every possible i/o case. We illustrate this point in Section 3 below.

2 Passive state/signal systems

We now proceed to study passivity in more detail, and the first step is to define what passivity means in the s/s setting. We therefore need to turn the ambient space $\left[\begin{smallmatrix}\mathcal{X}\\ \mathcal{X}\\ \mathcal{W}\end{smallmatrix}\right]$ into a Kreĭn space by introducing an indefinite inner product that measures the power lost by the system at any given time.

Definition 2.1. Let $\mathcal{X}$ be a Hilbert space with inner product $(\cdot,\cdot)_{\mathcal{X}}$ and let $\mathcal{W}$ be a Kreĭn space with indefinite inner product $[\cdot,\cdot]_{\mathcal{W}}$. The *node space* is $\mathfrak{K} := \left[\begin{smallmatrix}\mathcal{X}\\ \mathcal{X}\\ \mathcal{W}\end{smallmatrix}\right]$ equipped with the sesquilinear *power product*

$$\left[\begin{bmatrix} z^1\\ x^1\\ w^1\end{bmatrix}, \begin{bmatrix} z^2\\ x^2\\ w^2\end{bmatrix}\right]_{\mathfrak{K}} := [w^1, w^2]_{\mathcal{W}} - (z^1, x^2)_{\mathcal{X}} - (x^1, z^2)_{\mathcal{X}}. \tag{2.1}$$

A s/s node $(V;\mathcal{X},\mathcal{W})$ (and the system it induces) is *passive* if V is a maximally non-negative subspace of $\mathfrak{K}$. By this we mean that $[v,v]_{\mathfrak{K}} \geq 0$ for all $v \in V$ and that V has no proper extension which preserves this property.

The s/s node is *conservative* if V coincides with its *orthogonal companion*

$$V^{[\perp]} := \{v' \in \mathfrak{K} \mid \forall v \in V: \; [v, v'] = 0\}. \quad \blacklozenge$$

It follows from standard Kreĭn-space theory that every conservative s/s system is also passive. The condition that V is nonnegative is equivalent to the condition that every classical trajectory of a passive s/s system satisfies the inequality

$$[w(s), w(s)]_{\mathcal{W}} - (\dot{x}(s), x(s))_{\mathcal{X}} - (x(s), \dot{x}(s))_{\mathcal{X}} \geq 0, \quad s > 0.$$

Integrating this from 0 to t one obtains the equivalent condition that

$$\|x(t)\|_{\mathcal{X}}^2 \leq \|x(0)\|_{\mathcal{X}}^2 + \int_0^t [w(s), w(s)]_{\mathcal{W}} \,\mathrm{d}s, \quad t \geq 0, \tag{2.2}$$

and this inequality extends to generalised trajectories as well. The *maximal* nonnegativity ensures that there are enough trajectories in the sense that the s/s system has an i/s/o representation, cf. Theorem 3.1 below.

If a s/s node $\Sigma = (V;\mathcal{X},\mathcal{W})$ has an admissible i/o pair then the *s/s dual* $\Sigma^d = (V^{[\perp]};\mathcal{X},\mathcal{W})$ is a *time-reflected s/s system.* This means that the trajectories of Σ^d evolve as t tends from 0 to $-\infty$. More precisely, if we denote the space of generalised trajectories generated by $(V^{[\perp]};\mathcal{X},\mathcal{W})$ on $\mathbb{R}^-$

by $\mathfrak{W}^d$ and the function-reflection operator about 0 by $\mathbf{\mathfrak{R}}$, then $(\mathbf{\mathfrak{R}}\mathfrak{W}^d, \mathcal{X}, \mathcal{W})$ is the s/s system induced by the s/s node $(V'; \mathcal{X}, \mathcal{W})$, where

$$V' = \left\{ \begin{bmatrix} -z \\ x \\ w \end{bmatrix} \,\middle|\, \begin{bmatrix} z \\ x \\ w \end{bmatrix} \in V^{[\perp]} \right\}.$$

Remark 2.2. In particular, a s/s system is conservative if and only if it coincides with its own s/s dual, and these systems can thus be solved both in forward and backward time. See [Kur10a, Thm 4.11] for more details. ♦

If we drop the maximality condition then there is in general no guarantee that i/s/o representations exist. For example, $(\{0\}\,; \mathcal{X}, \mathcal{W})$ is a s/s node with the property that $[v, v]_{\mathfrak{K}} = 0$, $v \in V$. The s/s dual of this node is $(\mathfrak{K}; \mathcal{X}, \mathcal{W})$, and since $\mathfrak{K}$ is totally unstructured it has no meaning as a s/s system. In particular, $\mathfrak{K}$ does not satisfy condition (i) of Definition 1.1 if $\mathcal{X} \neq \{0\}$.

3 Input/state/output passivity

Every Kreĭn space $\mathcal{W}$ has a *fundamental decomposition* $\mathcal{W} = \mathcal{W}_+ \dotplus \mathcal{W}_-$, i.e., a direct-sum decomposition such that $\mathcal{W}_+$ is a Hilbert space, $\mathcal{W}_-$ is an anti-Hilbert space, and $[w_+, w_-]_{\mathcal{W}} = 0$ for all $w_\pm \in \mathcal{W}_\pm$. For every such decomposition we have

$$[w_+ + w_-, w_+ + w_-]_{\mathcal{W}} = \|w_+\|^2_{\mathcal{W}_+} - \|w_-\|^2_{|\mathcal{W}_-|}, \quad w_\pm \in \mathcal{W}_\pm, \tag{3.1}$$

where $\|w_-\|^2_{|\mathcal{W}_-|} = -[w_-, w_-]_{\mathcal{W}_-}$ is a Hilbert-space norm.

The following theorem collects some of the results in [Kur10a].

Theorem 3.1. *Let $\mathcal{X}$ be a Hilbert space, let $\mathcal{W}$ be a Kreĭn space, and assume that V is a maximally non-negative subspace of $\mathfrak{K}$ with the property (i) in Definition 1.1. Then $\Sigma = (V; \mathcal{X}, \mathcal{W})$ is a passive s/s system.*

Moreover, every fundamental decomposition $\mathcal{W} = \mathcal{W}_+ \dotplus \mathcal{W}_-$ induces an admissible i/o pair $(\mathcal{W}_+, \mathcal{W}_-)$ for a passive s/s system Σ, and the corresponding i/s/o representation is scattering passive. Indeed, from (2.2) *and* (3.1) *it follows that all classical trajectories with $u(t) \in \mathcal{W}_+$ and $y(t) \in \mathcal{W}_-$ satisfy*

$$\|x(t)\|^2_{\mathcal{X}} + \int_0^t \|y(s)\|^2_{|\mathcal{W}_-|}\, \mathrm{d}s \leq \|x(0)\|^2_{\mathcal{X}} + \int_0^t \|u(s)\|^2_{\mathcal{W}_+}\, \mathrm{d}s, \quad t \geq 0. \tag{3.2}$$

If Σ is conservative then the i/s/o representation is scattering conservative.

Conversely, let $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$ *be a scattering passive (conservative) i/s/o system with state space* $\mathcal{X}$*, input space* $\mathcal{U}$*, and output space* $\mathcal{Y}$*, all Hilbert spaces. Set* $\mathcal{W} := \left[\begin{smallmatrix} \{0\} \\ \mathcal{U} \end{smallmatrix}\right] \dot{+} \left[\begin{smallmatrix} \mathcal{Y} \\ \{0\} \end{smallmatrix}\right]$*, with inner product*

$$\left[\left[\begin{array}{c} y^1 \\ u^1 \end{array}\right], \left[\begin{array}{c} y^2 \\ u^2 \end{array}\right]\right]_{\mathcal{W}} := (u^1, u^2)_{\mathcal{U}} - (y^1, y^2)_{\mathcal{Y}}.$$

Then $\mathcal{W}$ *is a Kreĭn space with fundamental decomposition* $\mathcal{W} := \left[\begin{smallmatrix} \{0\} \\ \mathcal{U} \end{smallmatrix}\right] \dot{+} \left[\begin{smallmatrix} \mathcal{Y} \\ \{0\} \end{smallmatrix}\right]$*, and defining* V *by* (1.2)*, we obtain a passive (conservative) s/s node* $(V; \mathcal{X}, \mathcal{W})$ *with i/s/o representation* $\left(\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]; \mathcal{X}, \left[\begin{smallmatrix} \{0\} \\ \mathcal{U} \end{smallmatrix}\right], \left[\begin{smallmatrix} \mathcal{Y} \\ \{0\} \end{smallmatrix}\right]\right)$*.*

Since every scattering-passive i/s/o system is well-posed, it follows that every passive s/s system is a well-posed s/s system.

Now assume that $\mathcal{W} = \mathcal{U} \dot{+} \mathcal{Y}$ is a *Lagrangian decomposition* of $\mathcal{W}$, i.e., that $\mathcal{U} = \mathcal{U}^{[\perp]}$ and $\mathcal{Y} = \mathcal{Y}^{[\perp]}$. Then there exist Hilbert-space inner products on $\mathcal{U}$ and $\mathcal{Y}$, and a unitary operator $\Psi : \mathcal{U} \to \mathcal{Y}$, such that the Kreĭn-space inner product on $\mathcal{W}$ is given by

$$\left[\left[\begin{array}{c} y^1 \\ u^1 \end{array}\right], \left[\begin{array}{c} y^2 \\ u^2 \end{array}\right]\right]_{\mathcal{W}} = (y^1, \Psi u^2)_{\mathcal{Y}} + (\Psi u^1, y^2)_{\mathcal{Y}}. \tag{3.3}$$

Combining (2.2) and (3.3), we obtain that the following inequality holds for every trajectory of a passive state/signal node ($u(s) \in \mathcal{U}$ and $y(s) \in \mathcal{Y}$):

$$\|x(t)\|_{\mathcal{X}}^2 \le \|x(0)\|_{\mathcal{X}}^2 + 2\mathrm{Re} \int_0^t (\Psi u(s), y(s))_{\mathcal{Y}} \, \mathrm{d}s, \quad t \ge 0. \tag{3.4}$$

We have the following variation of Theorem 3.1 for Lagrangian decompositions. Unlike fundamental decompositions, Lagrangian decompositions do not always exist, and if they do, then they need not be admissible for passive s/s systems. Even if a Lagrangian decomposition is admissible, the corresponding i/s/o representation may not be well-posed.

Proposition 3.2. *Let* $\Sigma = (V; \mathcal{X}, \mathcal{W})$ *be a passive s/s system. If the Lagrangian decomposition* $\mathcal{W} = \mathcal{U} \dot{+} \mathcal{Y}$ *is admissible for* Σ *then the corresponding i/s/o representation is impedance passive, since all its classical trajectories satisfy* (3.4)*. If* Σ *is conservative then the i/s/o representation is impedance conservative.*

Conversely, let $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$ *be an impedance passive (conservative) i/s/o system with state space* $\mathcal{X}$*, input space* $\mathcal{U}$*, and output space* $\mathcal{Y}$*, all Hilbert spaces. Set* $\mathcal{W} := \left[\begin{smallmatrix} \{0\} \\ \mathcal{U} \end{smallmatrix}\right] \dot{+} \left[\begin{smallmatrix} \mathcal{Y} \\ \{0\} \end{smallmatrix}\right]$*, with inner product* (3.4)*. Then* $\mathcal{W}$ *is a Kreĭn space, and defining* V *by* (1.2)*, we obtain a passive (conservative) s/s node* $\Sigma = (V; \mathcal{X}, \mathcal{W})$*. Furthermore, the Lagrangian i/o pair* $\left(\left[\begin{smallmatrix} \{0\} \\ \mathcal{U} \end{smallmatrix}\right], \left[\begin{smallmatrix} \mathcal{Y} \\ \{0\} \end{smallmatrix}\right]\right)$ *is admissible for* Σ *and the corresponding i/s/o representation is* $\left[\begin{smallmatrix} A\&B \\ C\&D \end{smallmatrix}\right]$*.*

The energy inequalities (3.2) and (3.4) correspond to the fundamental and Lagrangian decompositions of $\mathcal{W}$, respectively, but the *property of passivity* is characterised by the maximal non-negativity of V. Thus passivity is an *input/output invariant* property of the state/signal node. Moreover, a passive s/s system is characterised by the fact that V is a maximally non-negative subspace of $\mathfrak{K}$ with the property (i) in Definition 1.1. Comparing this to the rather complicated definition of an i/s/o system node, we see that the s/s definition is much simpler, in addition to being more general.

4 Port-Hamiltonian systems

In the second and shorter part of the thesis I discuss port-Hamiltonian systems in the Hilbert space setting. Port-Hamiltonian systems have their historical roots in energy-based modelling and control of physical systems, mainly non-linear finite-dimensional systems, and the theory has turned out to be useful in applications; see for example the energy-efficient walking robots developed by Duindam and Stramigioli [DS09].

A port-Hamiltonian system essentially consists of two parts: the Hamiltonian $\mathcal{H}$, which measures the total energy of the system when it is in a given state x, and the Dirac structure $\mathcal{D}$. In the thesis only a quadratic Hamiltonian of the type

$$\mathcal{H}(x(t,\cdot)) = \frac{1}{2} \int\limits_{z\in\Omega} \|x(t,z)\|^2 \,\mathrm{d}z$$

is considered, where Ω is the domain of the underlying partial differential equation, because I only work with *linear systems*, but one can also use non-quadratic Hamiltonians. The Dirac structure encodes the relations between the system variables and it is essentially the port-Hamiltonian equivalent of the generating subspace V.

The Dirac structure also describes how the system acts when it is interconnected with another port-Hamiltonian system. Interconnection is a fundamental operation in control theory, since plants are controlled by interconnection with controllers. In the Port-Hamiltonian context, interconnection is a generalisation of the feedback control found in classical control theory. In the thesis I describe how interconnection of port-Hamiltonian systems is carried out by composing their respective Dirac structures.

The article [KZvdSB10] aims at giving useful necessary and sufficient conditions for the composition of two Dirac structures to be a Dirac structure, i.e., for the energy-preserving interconnection of two port-Hamiltonian systems to be a port-Hamiltonian system. In the finite-dimensional case the composition of two Dirac structures is always a Dirac structure, but this question is non-trivial in the infinite-dimensional setting.

The theoretical concepts discussed in the thesis are throughout illustrated on the example of an ideal transmission line. In particular, I give an example from [KZvdSB10], where the Telegrapher's equations that describe the dynamics of the transmission line are turned into a Schrödinger equation by composition with another Dirac structure along the spatial domain $\Omega = (0, \infty)$. On a more abstract level, the same example connects the port-Hamiltonian system to its close relative, the conservative s/s system.

References

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