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EXPERIMENTAL EVALUATION OF INPUT DESIGNS FOR MULTIPLE-INPUT MULTIPLE-OUTPUT OPEN-LOOP SYSTEM IDENTIFICATION

Kurt E. Häggblom, Jari M. Böling
Åbo Akademi University, Department of Chemical Engineering
Biskopsgatan 8, FI-20500 Turku, Finland
khaggblo@abo.fi, jboling@abo.fi

ABSTRACT
A procedure is presented for input design in MIMO system identification that explicitly takes system gain directionality into account. For ill-conditioned systems, the dynamics tend to be different in the various gain directions. The advantage of this procedure is that the dynamics in all gain directions can be identified. The procedure can be used for any type of excitation signal, e.g., step, PRBS or multi-sinusoidal signals. The superiority of the proposed input designs over more standard designs is demonstrated on a pilot-scale distillation column. The main conclusion of the study is that it is crucial that the various gain directions in a MIMO system are properly excited. The type of input signal (e.g., step or PRBS), or the way of exciting the gain directions, appears to be less important.

KEY WORDS
System identification, Experiment design, Multivariable systems, Ill-conditioned systems, Distillation columns, PRBS signals.

1. Introduction

The directionality properties of ill-conditioned multiple-input multiple-output (MIMO) systems make the identification task much harder than for single-input single-output systems. In the research community it is well established that a proper identification requires that the various directions are explicitly excited by the inputs (see [1]–[6]). Usually, PRBS signals are used as perturbations, but there is also a certain trend towards more "plant-friendly" signals such as multi-sinusoidal signals [6], [7]. Even simple, properly designed step changes can be used for excitation of the relevant directions [2].

In practice, it is quite common to perturb the inputs one at a time, or simultaneously by uncorrelated PRBS signals. However, this does not properly excite the various gain directions of the system, especially not the low-gain direction. An interesting question is to what extent, if any, the directionality problems can be reduced by the use of an "active", but suboptimally implemented, perturbation such as a PRBS signal as compared to, e.g., optimally designed step perturbations. This is one of the issues, which in this paper are investigated experimentally on a pilot-scale distillation column.

We also present a simple, but general, procedure for input design in MIMO system identification that explicitly takes the directionality issue into account. The procedure can be used for any type of excitation signal, e.g., step, PRBS and (multi)sinusoidal signals. In this work, the procedure is applied to the design of identification experiments using step changes and PRBS signals. The superiority of the designs over more standard designs is demonstrated by cross validations of model predictions.

2. Input Design for MIMO Identification

2.1 Effects of Ill-Conditioning

A system becomes ill-conditioned, when (some of) its outputs are almost linearly interdependent or, equivalently, (some of) its inputs have nearly identical effects. This means that the gain matrix between the outputs and the inputs has rows and columns that are almost linearly interdependent. Such a matrix has a high condition number and is nearly singular.

The consequences of this can be quite dramatic. Consider

$$y = Gu\,, \quad G = \begin{bmatrix} 0.505 & -0.495 \\ 0.495 & -0.505 \end{bmatrix}, \tag{1}$$

where y is a vector of outputs, u is a vector of inputs, and G is a gain matrix. This matrix has the condition number 100 (the singular values are 1 and 0.01), which is not excessively large for an ill-conditioned system.

As revealed by the gain matrix, it is "easy" to change the outputs in the same direction. For example, the input $u = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}}$ yields $y = \begin{bmatrix} 0.505 & 0.495 \end{bmatrix}^{\mathrm{T}}$. It is much "harder" to change the outputs in opposite directions. If we desire $y = \begin{bmatrix} 0.505 & -0.495 \end{bmatrix}^{\mathrm{T}}$, $u \approx \begin{bmatrix} 50.0 & 50.0 \end{bmatrix}^{\mathrm{T}}$ is required, i.e., inputs that are about 50 times larger than the inputs in the easy case. However, if we apply the input $u = \begin{bmatrix} 50.5 & 49.5 \end{bmatrix}^{\mathrm{T}}$ instead, e.g., due to some small input inaccuracy, we get $y = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}}$, which is completely different from the desired output.

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In this example, the input $u = [0.7 \;\; -0.7]^{\mathrm{T}}$, with the 2-norm $\| u \| = 1$, gives the strongest output amplification, i.e., $y = [0.7 \;\; 0.7]^{\mathrm{T}}$ with $\| y \| = 1$. This input direction is called the high-gain direction. The smallest gain is obtained by $u = [0.7 \;\; 0.7]^{\mathrm{T}}$, which gives the output $y = [0.007 \;\; 0.007]^{\mathrm{T}}$ with $\| y \| = 0.01$. This direction is called the low-gain direction.

The dynamics of an ill-conditioned system often complicates the issue [2], [8]. Consider the second-order transfer function

$$G = \frac{K}{(T_1 s + 1)(T_2 s + 1)} = \frac{K_1}{T_1 s + 1} + \frac{K_2}{T_2 s + 1} . \tag{2}$$

In an ill-conditioned MIMO system like a distillation column, every transfer function in the transfer gain matrix between the quality outputs (e.g., product concentrations) and the manipulated internal flow rates is approximately of second order with one large time constant T_1 and one small time-constant T_2. Moreover, the gain K_1 in every transfer function is a gain in (or close to) the high-gain direction, and the gain K_2 is a gain in (or close to) the low-gain direction. In standard identification procedures, it is difficult to determine T_2 if it is much smaller than T_1. However, if can excite only the low-gain direction as described in [2], it becomes much easier to determine T_2.

The behaviour of an ill-conditioned system resembles that of a strongly nonlinear system although it is linear. Furthermore, the system is very sensitive to (input) uncertainty. These issues make control very difficult. However, they also make identification and modelling demanding tasks due to the high accuracy required.

2.2 Design Principle for Open-Loop Identification

A successful identification of an ill-conditioned system requires that the various directions are properly excited. It is easy to obtain information about the high-gain direction, but in order to obtain information about the low-gain direction, it must be explicitly excited. Otherwise, the obtained model will predict low-gain properties poorly and it may be inadequate for control design [2].

A proper excitation requires that all inputs are perturbed simultaneously and they have to be correlated in certain ways. To see this, consider a system with the input vector u, the output vector y, and the transfer matrix G. A singular value decomposition (SVD) of the transfer matrix gives

$$y = Gu = U \Sigma V^{\mathrm{T}} u , \tag{3}$$

where U and V are unitary matrices and Σ is a diagonal matrix of singular values, σ_i, $i = 1, \ldots, n$. Let us denote the ith vectors of U and V by U_i and V_i, respectively. The input $u = u^i = \sigma_i^{-1} V_i$ then produces the output $y = y^i = U_i$, which is the ith input/output/gain direction. The scaling of u^i by $k\sigma_i^{-1}$ means that all directions give the same output norm $\| y^i \| = \| kU_i \| = k$. This is a desirable feature that makes the outputs in all directions equally informative.

To properly excite all directions i, $i = 1, \ldots, n$, we need to apply inputs u^i that vary (symmetrically) between the limits

$$u_-^i = -\sigma_i^{-1} V_i \;\; , \quad u_+^i = +\sigma_i^{-1} V_i \;\; , \quad i = 1, \ldots, n . \tag{4}$$

This can be done in several ways. One can, e.g., us series of step changes, PRBS signals, or more "plant-friendly" multi-sinusoidal signals [6], [7]. One can choose to excite only one direction at a time, or to excite all directions simultaneously. In the latter case,

$$u = \frac{1}{n} \sum_{i=1}^{n} u^i , \tag{5}$$

where each input direction u^i is driven by separate, mutually uncorrelated, signals.

Note that the requirement to excite all gain directions cannot be circumvented by closed-loop identification [3], where the setpoint of one output at a time is changed. In order to determine the dynamics, especially small time constants associated with low-gain directions, the gain directions have to be excited also in closed-loop identification. The outputs should then be perturbed simultneously along the output directions indicated by U, i.e., $y = y^i = U_i$, $i = 1, \ldots, n$.

It is possible to design the signals for the various directions with different dynamics in mind. This may be useful, e.g., in the identification of distillation columns since it has been observed in simulation studies that the high-gain dynamics are relatively slow whereas the low-gain dynamics can be quite fast [8].

Obviously, approximate knowledge of σ_i is sufficient for calculation of an adequate input u^i, because σ_i does not affect the direction. However, it is more important to know V_i accurately, as illustrated in the previous section. For distillation columns, it is fortunate that these directions can be accurately estimated from certain flow gains that are easy to determine in practice [9].

3. Experimental Application

In this section, the usefulness of the suggested open-loop identification method is illustrated by an application to a pilot-scale distillation column. The distillation column is 30 cm in diameter, has 15 bubble-cap trays, and separates a mixture of water and ethanol. In addition to tray temperatures, the distillate and bottom product compositions are measured on-line. Feed, product, reflux and steam flow rates are manipulated by pumps and valves. The feed tank capacities suffice for 8 hour continuous operation. A picture of the distillation column is shown in Fig. 1.

3.1 Experiment Designs

As far as step changes are concerned, the importance of the design principles laid out above have been verified in a previous study using the distillation column mentioned above [2]. An interesting question is to what extent, if any, the directionality problems can be reduced by the use of a more sophisticated type of input perturbation such as PRBS signals. We shall here investigate this question as well as the various ways of taking directionality into account. For purposes of comparison, we also repeat the type of experiments done in [2].

The following input designs are considered:

1. A sequence of step changes in inputs, one at a time (`SeqStep`).
2. A sequence of step changes in low- and high-gain directions (`DirStep`).
3. PRBS signals in inputs, one at a time (`SeqPRBS`).
4. Simultaneous uncorrelated PRBS signals (`UncPRBS`).
5. PRBS excitation of low- and high-gain directions, one at a time (`SeqDirPRBS`).
6. Simultaneous PRBS excitation of low- and high-gain directions (`SimDirPRBS`).

In the design of PRBS signals, guidelines given in [10] were used as a starting point. However, major constraints were the capacity of the feed tanks and the need to reach an approximate steady state after start-up before an experiment could be started. Such considerations and the fact that the dominating time constant of the distillation column is about 20 min, motivated a switching time $T_{sw} = 5$ min and a sequence length $N = 63$ (`UncPRBS` and `SimDirPRBS`) or two sequences of length $N = 31$ (`SeqPRBS` and `SeqDirPRBS`).

Fig. 1. Pilot-scale distillation column at ÅAU.

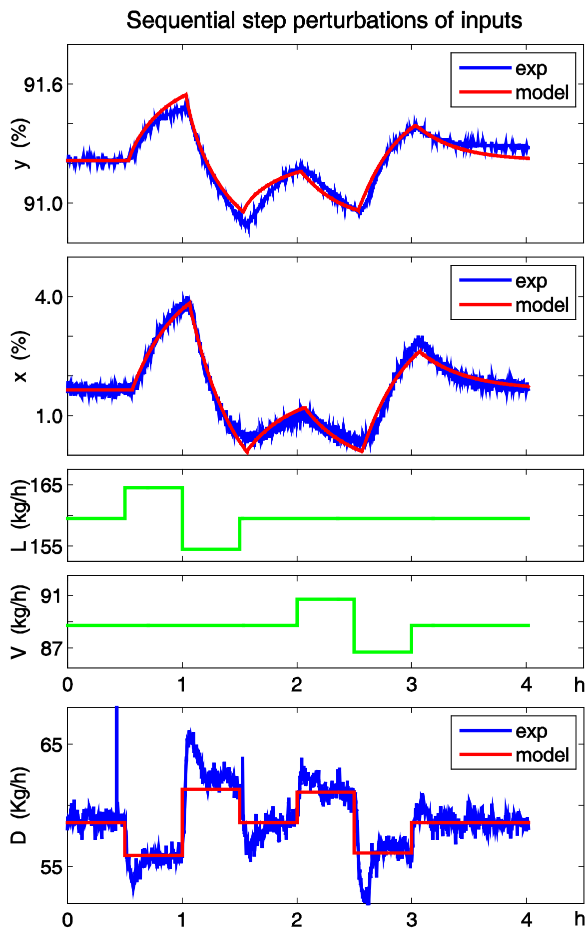

Fig. 2. Sequential step changes of inputs (`SeqStep`).

3.2 Experiments and Model Fits

In all cases, each input-output relationship was modelled as a second-order transfer function with a time delay. Because most experiments were affected by drift in data, which was difficult to quantify due to slightly non-stationary conditions, outputs were detrended simultaneously with the fitting of model outputs to data [11]. Only detrended data are shown in the graphs that follow.

3.2.1 Sequential step changes

In the experiment shown in Fig. 2 (`SeqStep`), a series of step changes is applied to the inputs, i.e., the reflux flow L and the steam flow V to the reboiler, one at a time. The main outputs are the distillate composition y and the bottoms composition x. Included is also the distillate flow rate D, which controls the holdup in the overhead condenser drum. The gains between the distillate flow rate and the inputs, which can easily be determined from the data, are useful because they make it possible to estimate the low- and high-gain directions needed for the design of experiments, where these directions are explicitly excited.

The model outputs are also included in Fig. 2. The fits look reasonable good, although there appears to be some difficulty with nonlinearity.

3.2.2 Directional step changes

Figure 3 shows an experiment (`DirStep`), where the inputs are changed simultaneously, first in the low-gain direction, then in the high-gain direction. Even though the input changes in the high-gain direction in the latter part of the experiment are very small, the effect on the outputs is as strong as the much larger input changes in the low-gain direction in first part of the experiment. These input directions are calculated from the flow gains obtained from the previous experiment as in [2] and [9].

The model fits are quite good, even the fit of the bottoms composition, although heavy detrending of these data were needed.

3.2.3 Sequential PRBS signals

Figure 4 shows the first of four experiments, where PRBS signals are used. In this experiment (`SeqPRBS`), the reflux flow is perturbed in the first part, the reboiler steam flow in the latter part.

The model fits look reasonable good, although there are some larger deviations in the middle and at the end.

3.2.4 Uncorrelated PRBS signals

In Fig. 5, both inputs are perturbed simultaneously by statistically uncorrelated PRBS signals (`UncPRBS`). This is achieved by suitably time-shifting two identical PRBS signals [7]. This is probably the most used "advanced" input design for MIMO system identification.

As shown, the model fits appear to be very good.

3.2.5 Directional PRBS signals in sequence

The experiment in Fig. 6 is the first of two experiments, where the PRBS inputs are designed to explicitly excite

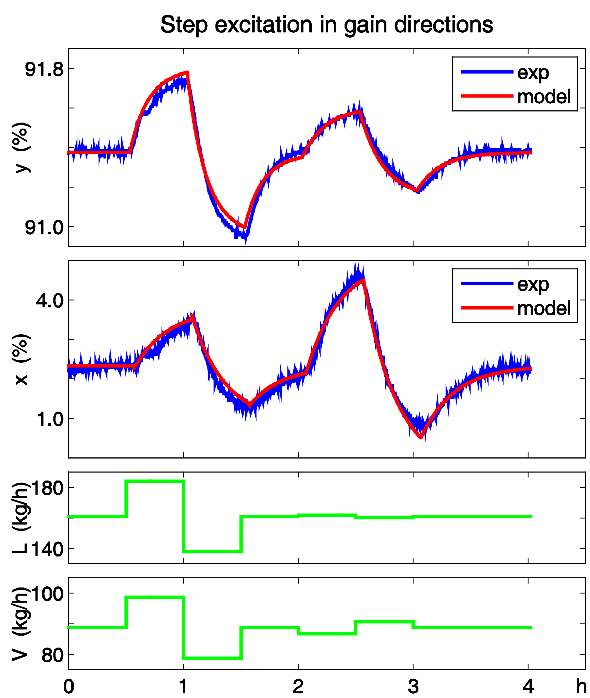

Fig. 3. Step perturbations in gain directions (`DirStep`).

the low- and high-gain directions. In this experiment (`SeqDirPRBS`), the low gain direction is excited in the first part, and the high-gain direction in the latter part. Again, the strong effect of the small input perturbations in the high-gain direction is notable.

The model fit of the distillate composition is excellent, but the bottoms composition fit is not quite as good.

3.2.6 Directional PRBS signals simultaneously

In Fig. 7, the low- and high-gain directions are excited simultaneously by two uncorrelated, superimposed, PRBS signals, one exciting the low-gain direction, the other exciting the high-gain direction (`SimDirPRBS`).

Here, too, the model fit of the distillate composition is excellent, but the fit of the bottoms composition is

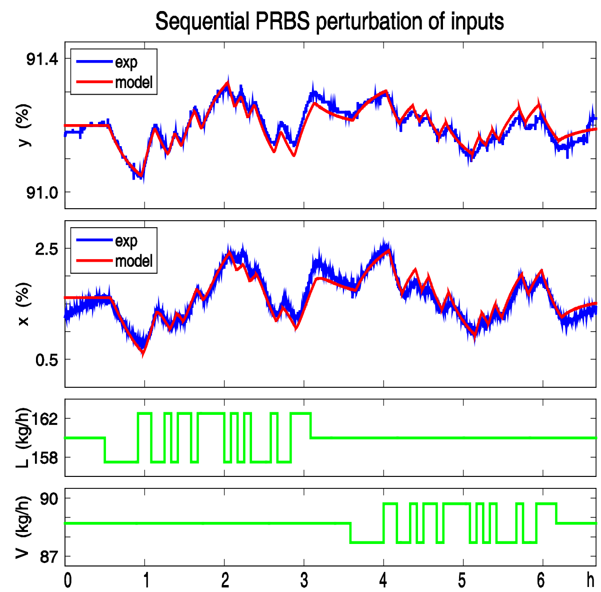

Fig. 4. Sequential PRBS perturbations (`SeqPRBS`).

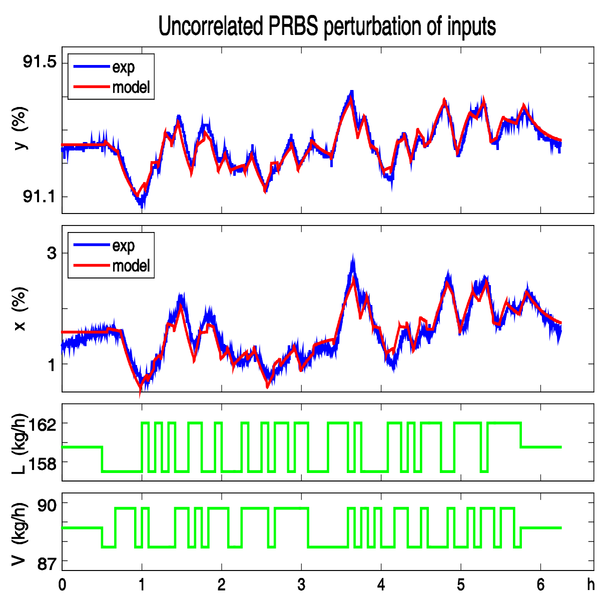

Fig. 5. Uncorrelated PRBS perturbations (`UncPRBS`).

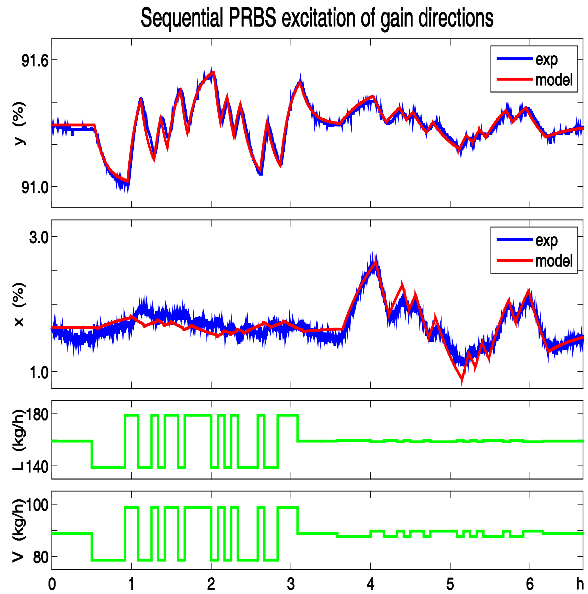

Fig. 6. Sequential PRBS excitation of gain directions (`SeqDirPRBS`).

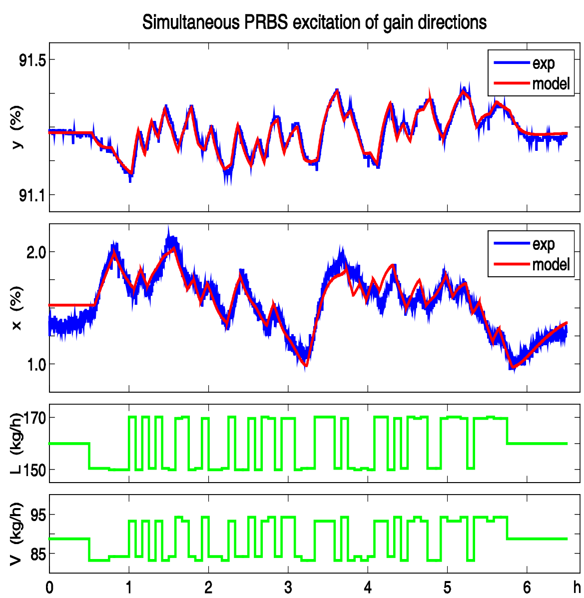

Fig. 7. Simultaneous PRBS excitation of gain directions (`SimDirPRBS`).

somewhat worse. However, the bottoms composition is much more contaminated by noise than the distillate composition. Furthermore, there is a clear transient in the bottoms composition in the beginning of the experiment.

3.3 Cross Validations

Next, the quality of the models, and thus the quality of the data obtained by the various experimental designs, is illustrated by cross validations by testing how the models obtained by various experiments can predict the outcome of other experiments.

3.3.1 Sequential step changes

Figure 8 shows how the model obtained from the experiment with step changes in the inputs, one at a time (`SeqStep`), predicts the distillate output for other experiments, where directionality was not taken into account (`SeqPRBS` and `UncPRBS`). The model fit is repeated in the uppermost graph. Figure 9 shows how the same model predicts data obtained by exciting the gain directions (`DirStep`, `SeqDirPRBS`, `SimDirPRBS`). As can be seen, these predictions are worse than the previous ones.

3.3.2 Uncorrelated PRBS signals

Figure 10 shows how the model obtained by simultaneous uncorrelated PRBS signals (`UncPRBS`) predicts data of other non-directional experiments (`SeqStep`, `SeqPRBS`), whereas Fig. 11 shows data predictions of directional experiments (`DirStep`, `SeqDirPRBS`, `SimDirPRBS`). The predictions of directional data is worse than the predictions by the model obtained from `SeqStep` data.

3.3.3 Directional step changes

Figure 12 and 13 show predictions by the model obtained from step changes in the low- and high-gain directions (`DirStep`). The predictions of non-directional (`SeqStep`, `SeqPRBS`, `UncPRBS`) as well as directional (`SeqDir-PRBS`, `SimDirPRBS`) excitation data are excellent.

3.3.4 Directional PRBS signals simultaneously

Figure 14 and 15, finally, show the performance of the model obtained by simultaneous, but explicit, excitation of the low- and high-gain directions by PRBS signals (`SimDirPRBS`). Here too, the predictions of both non-directional (`SeqStep`, `SeqPRBS`, `UncPRBS`) and directional data (`DirStep`, `SeqDirPRBS`) are good.

4. Conclusion

We have outlined a procedure for input design in MIMO system identification that explicitly takes into account the directionality caused by ill-conditioning of a strongly interactive process. The procedure can be used for any type of excitation signal, e.g., step, PRBS and multi-sinusoidal signals. In this work, the procedure was applied to the design of informative identification experiments employing step changes and PRBS signals.

The superiority of the designs over more standard designs was demonstrated on a pilot-scale distillation column. It was shown by cross validations that experiments, where the directionality issues were not addressed, resulted in models that were less successful at predicting outputs of other experiments than the models obtained from experiments designed to explicitly take directionality into account. In this respect, step changes in the low- and high-gain directions was a better input design than simultaneous, uncorrelated, PRBS signals. For the same type of input designs, the difference between step

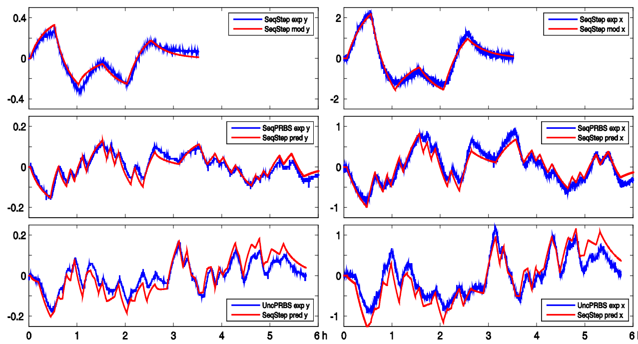

Fig. 8. `SeqStep` predictions of non-directional data.

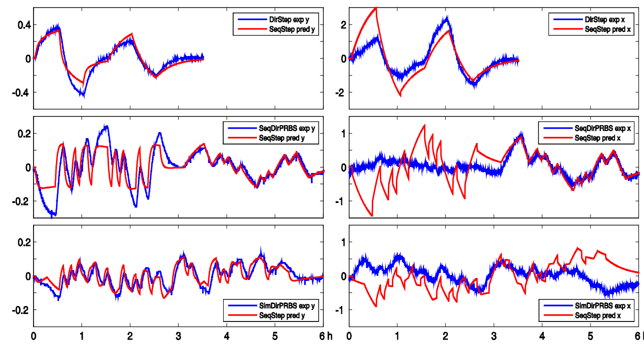

Fig. 9. `SeqStep` predictions of directional data.

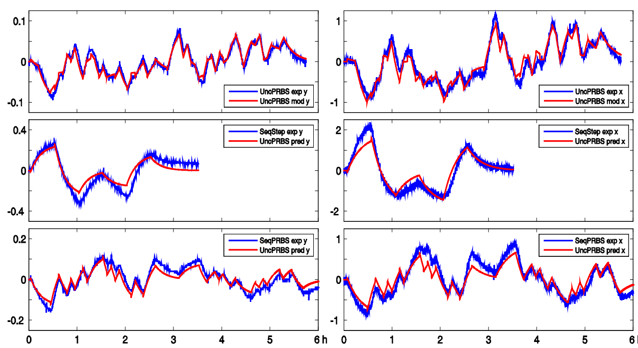

Fig. 10. `UncPRBS` predictions of non-directional data.

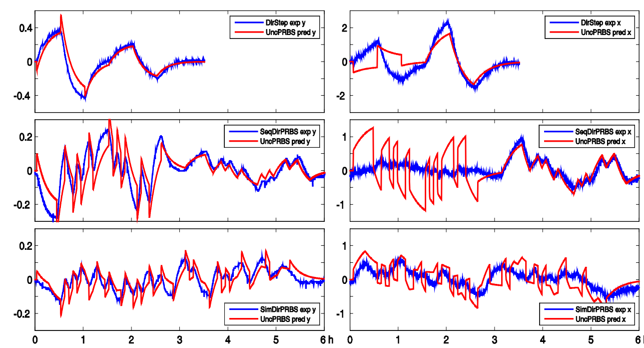

Fig. 11. `UncPRBS` predictions of directional data.

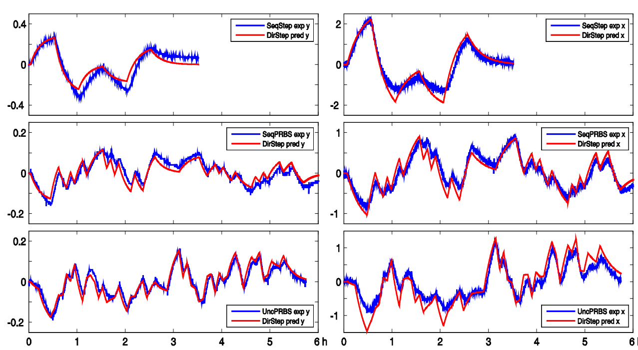

Fig. 12. `DirStep` predictions of non-directional data.

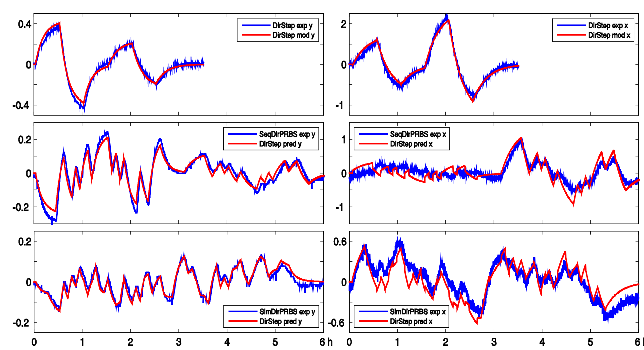

Fig. 13. `DirStep` predictions of directional data.

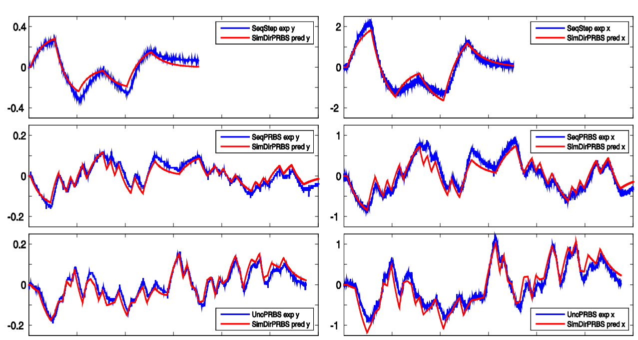

Fig. 14. `SimDirPRBS` predictions of non-directional data.

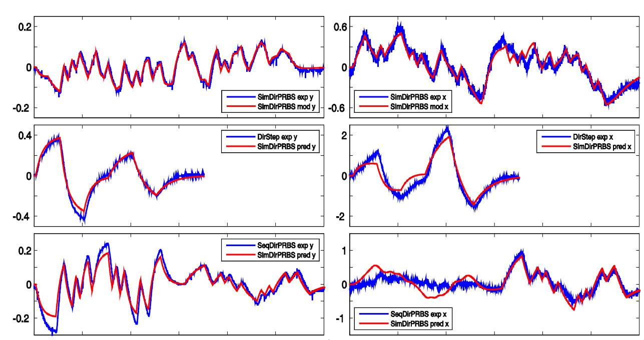

Fig. 15. `SimDirPRBS` predictions of directional data.

changes and PRBS signals was small. However, although not an outcome of this study, identification experiments using PRBS signals can be expected to be less sensitive to disturbances than experiments with step changes.

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