Published in IET Control Theory and Applications
Received on 18th December 2007
Revised on 11th June 2008
doi: 10.1049/iet-cta:20070479


ISSN 1751-8644

# Model-based torsional vibration control of internal combustion engines

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**Abstract:** For internal combustion engines, it is important to ensure uniform cylinder-wise torque contributions in order to avoid excessive crankshaft torsional vibrations. Especially, the high-torsional vibration levels of medium-speed power plants and marine engines cause unnecessary wear of mechanical components, such as the flexible coupling between the engine and the load. This is because of the fact that the lower torque-order frequencies excited by the fuel combustions are usually in the vicinity of the natural frequency of the flexible coupling. A cylinder-balancing method is presented, which minimises crankshaft torsional vibrations on medium-speed internal combustion engines. Using a model of engine dynamics, the reduction of crankshaft torsional vibrations can be addressed as an online optimisation problem, where the Newton direction of fuel-injection adjustments is determined at each step. The proposed method is tested on a six-cylinder 6 MW Wärtsilä power plant engine, showing that the torsional vibration level can be significantly reduced, well below admissible levels.

## 1 Introduction

The introduction of a common-rail fuel system for diesel engines has made it possible to control cylinder-wise fuel-injection durations and timings. As the fuel is stored under constant high pressure and distributed to the cylinders by means of fast-switching electromagnetic solenoid valves, optimal fuel combustion can be achieved regardless of the engine load, as opposed to conventional fuel systems. The opportunities for better control have resulted in significant reductions of emitted air pollutants and soot generation. Common-rail fuel systems are now well established in automotive applications, and have in recent years also been increasingly applied to heavy fuel medium-speed marine diesel engines, [1].

An important problem in the common-rail fuel system is the calibration of the fuel amounts injected into the individual cylinders. Because of the varying characteristics of the solenoid valves, oscillating pressure differences between the rail and cylinders or clogging of the injector nozzles, there may exist a considerable discrepancy between the demanded and actual fuel amount injected into the cylinders. In automotive applications, this discrepancy can reportedly be as high as 25% [2]. The resulting non-uniform cylinder-wise torque contributions cause considerable crankshaft torsional vibrations, increasing the wear of mechanical components. Although the automotive industry has addressed this problem by using automatic calibration methods, medium-speed marine engines are today calibrated manually by periodically adjusting the fuel-injection durations in such a way that the exhaust-gas temperatures, or the cylinder pressures are aligned. For engines with a large number of cylinders, this is a tedious and difficult task. Moreover, cylinder balancing based on temperature and pressure measurements does not guarantee that the torsional vibrations are minimised.

Balancing cylinder-wise torque contributions of internal combustion engines has in recent years been subject to active research, where various approaches have been proposed. These methods can be divided into two groups: estimation of the absolute cylinder-wise output power and estimation of the relative contributions of the cylinders. A frequently used class of methods is based on estimating the net indicated torque by using measurements of the angular acceleration of the flywheel, [3, 4]. Lee *et al.* [5] address the problem of estimating the cylinder pressures and gas torques using nonlinear parameter estimation. A set of nonlinear basis functions are used along with measurements of the crankshaft position, speed and acceleration to estimate the cylinder pressures. A nonlinear sliding-mode observer was suggested in [6] where a simplified state-space combustion model, including compression and expansion strokes, was utilised together with a crankshaft model to estimate the cylinder pressure. A comparison of different approaches was done in [7], where analysis of the angular position and acceleration of the crankshaft, as well as the exhaust manifold pressure were used for detection and correction of non-uniform torque contributions. A frequency-domain method to determine the relative torque contributions was proposed in [8]. In this approach, the cylinder contributing the most to the torsional vibrations is identified by analysing the measured engine speed by means of the engine-specific phase-angle diagrams, which relate the firing sequence of the engine to the phases of the frequency components excited by the fuel combustions, [9, 10]. In [11], another frequency-domain method has been suggested where the lower frequencies of, for example engine speed, exhaust-gas manifold pulsations or the turbocharger instantaneous speed are used to align the torque contributions of the cylinders.

The relative torque contribution profile of the cylinders can be determined from the superposed oscillating gas torque component applied on the flywheel. This is done using measurements of the angular speed of the flywheel and a one-mass lumped model of the engine with compensation for mass, friction and valve torque, [2, 12–14]. For example, by calculating the change in the kinetic energy of the rotating crankshaft system and relating these changes to the consecutive cylinder firings, a measure of the cylinder-wise torque contributions is obtained, which can be exploited for cylinder balancing [2].

The methods which have been developed in automotive applications are based on a number of assumptions, such as the engine being dynamically sufficiently decoupled from the load, which are usually not valid for large medium-speed engines [15]. In [15], the approach suggested in [8] has been generalised to large medium-speed engines in marine and power installations by taking into account the dynamic link between the engine and the driveline. In this method, qualitative analysis of the superposed torque applied on the flywheel is used together with the phase-angle diagrams of the engine to identify the cylinder which deviates the most from the average torque output. The fuel injection of the cylinder identified in this way is then adjusted appropriately.

In the methods based on a qualitative analysis of the phase-angle diagrams [8, 15], a single cylinder is adjusted at each stage. This may, however, result in slow convergence and unnecessarily lengthy iterations in cases where more than one cylinder needs to be adjusted. In this paper, the cylinder balancing method for medium-speed engines presented in [15] is generalised by using a quantitative analysis of the phase-angle diagrams to facilitate the simultaneous adjustments of several cylinders. The proposed method is based on the online minimisation of a cost function, which consists of the amplitudes of the set of torsional vibration frequencies to be reduced as well as exhaust-gas temperature deviations.

The paper is organised as follows. The torsional vibration control problem is presented in Section 2, along with a description of the cylinder balancing method and the cost function used. Section 3 derives an engine model for reconstruction of the gas torque from angular speed measurements of the crankshaft, which is more appropriate for power plant or medium-speed marine engines than the automotive engine model which is commonly used in the literature. In Section 4, a model relating fuel-injection duration adjustments to the amplitudes and the phases of the reconstructed torque orders is developed. Section 5 presents a cylinder balancing control strategy to minimise torsional vibrations. Experimental results using the proposed cylinder balancing method on a six-cylinder 6 MW Wärtsilä power plant engine are finally presented in Section 7.

## 2 Problem formulation

The problem studied in this paper is to minimise the torsional vibration level of internal combustion engines for a number of specified torque-order frequencies by manipulating fuel injections. For a four-stroke engine, the set of vibration frequencies which can be affected is limited to the orders, or multipliers of the rotational frequency, $p = 1/2, 1, 3/2, 2, \ldots$ which are excited by the fuel combustions [9]. The main idea is that for a balanced engine with a rigid crankshaft, the superposed lower torque-order amplitudes are approximately zero. It follows that the cylinder-wise torque contributions can be balanced by eliminating the torsional vibration orders below the ignition order, [14].

For large medium-speed engines, the cylinder-wise exhaust-gas temperatures are usually measured to avoid burning of the exhaust-gas valves. Although the exhaust gas temperatures, in addition to the fuel-injection durations, also depend on fuel-injection timings, cooling water temperature, inlet air temperature and so on, the temperature measurements can be used as rough indicators of the cylinder-wise torque contributions. As large exhaust-gas temperature deviations should be avoided in practice, they will also be taken into account in the cylinder balancing studied here.

The objective of this paper is to develop an iterative procedure which minimises the amplitudes of a given set of torsional vibration orders of the crankshaft by adjusting the


doi: 10.1049/iet-cta:20070479

cylinder-wise fuel injections. In the proposed procedure, an engine model is used to reconstruct the gas torque $M$ generated by the fuel combustions from measurements of the angular speed $\dot{\varphi}_1$ of the crankshaft, and optionally the angular speed $\dot{\varphi}_2$ of the load, [15]. The cylinder balancing method determines the optimal fuel adjustment vector $\mathbf{\Delta u}$ using the reconstructed gas torque $M$ and the exhaust-gas temperatures $\mathbf{T}_{\mathbf{exh}}$. A block diagram of the controller scheme is shown in Fig. 1.

For a four-stroke engine, the gas torque generated by the cylinder firings consists of frequency orders $p$, which are integer multipliers of the fundamental frequency $f/2$, where $f$ is the rotational frequency. Hence, the gas torque $M_u(t)$ has the Fourier series expansion

$$M_u(t) = \sum_{p=0,\pm(1/2),\pm 1,\pm(3/2),\ldots} M_{\mathrm{p}}(\mathbf{u})\mathrm{e}^{j2\pi fpt} \qquad (1)$$

where $\mathbf{u}$ is a vector of cylinder injections and $M_{\mathrm{p}}(\mathbf{u})$ are complex numbers which define the magnitudes and phases of the torque orders $p$. The cylinder balancing control problem can then be formulated as a minimisation of the cost

$$h(\mathbf{u}) = \frac{1}{2}\sum_{p\in \mathrm{P}} w_p \left|M_{\mathrm{p}}(\mathbf{u})\right|^2 \qquad (2)$$

where $P$ is the set of torque orders whose amplitudes are to be reduced and $w_p$ are non-negative weights reflecting the relative importance of the torque orders.

To maintain a constant engine power output, the cost (2) should be minimised subject to the constraint

$$\sum_{i=1}^{N_{\mathrm{cyl}}} u_i = u_{\mathrm{tot}} \qquad (3)$$

where $N_{\mathrm{cyl}}$ is the number of cylinders, $u_i$ are cylinder-wise fuel-injection durations and $u_{\mathrm{tot}}$ the required total fuel-injection duration that depends on the engine load and is determined by the speed controller.

In addition to requiring that torsional vibration orders be minimised, large deviations in the cylinder-wise exhaust-gas temperatures $T_i(u_i)$ should be avoided, and they are

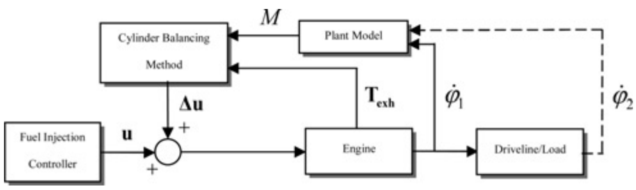


**Figure 1** *Control scheme*

therefore constrained according to

$$\left|T_i(u_i) - T_{\mathrm{a}}(\mathbf{u})\right|^2 \leq \Delta T_{\max}^2 \qquad (4)$$

where $\mathbf{u}$ is the fuel-injection duration vector and $T_{\mathrm{a}}(\mathbf{u})$ is the average exhaust-gas temperature

$$T_{\mathrm{a}}(\mathbf{u}) = \frac{1}{N_{\mathrm{cyl}}}\sum_{i=1}^{N_{\mathrm{cyl}}} T_i(u_i)$$

For large medium-speed diesel engines, the exhaust-gas temperatures are typically allowed to diverge maximally about 20°C. Notice that the constraints (3) and (4) are always achievable for internal combustion engines.

## 3 Engine model

To evaluate the cost (2), the superposed oscillating gas torque $M_u$ should be reconstructed from available angular speed measurements. In automotive applications, it is generally assumed that: (a) the engine is decoupled from the load, (b) the crankshaft is sufficiently rigid and (c) the load torque is constant, [16]. Under these assumptions the engine dynamics can be described by [12]

$$J_1(\varphi_1)\ddot{\varphi}_1 + \frac{1}{2}\frac{\mathrm{d}J_1(\varphi_1)}{\mathrm{d}\varphi_1}\dot{\varphi}_1^2 = M_u - M_{\mathrm{L}} - M_{\mathrm{F}}(\varphi_1) - M_{\mathrm{V}}(\varphi_1) \qquad (5)$$

where $J_1(\varphi_1)$ is the mass moment of inertia of the engine, $\ddot{\varphi}_1$ and $\dot{\varphi}_1$ the angular acceleration and speed of the crankshaft, respectively, $M_u$ the gas torque, $M_{\mathrm{L}}$ the load torque, $M_{\mathrm{F}}$ the friction torque and $M_{\mathrm{V}}$ the valve torque. Note that the mass moment of inertia and the mass torque, given by the term $(1/2)\cdot(\mathrm{d}J_1(\varphi_1)/\mathrm{d}\varphi_1)\dot{\varphi}_1^2$, depend on the crank angle, which should be taken into account when using angular speed measurements to reconstruct the superposed gas torque from (5), [10, 12].

The assumption that the engine is decoupled from the load is usually valid in automotive applications, since the nominal rotational speed of the engine is typically significantly higher than the first natural frequencies of the crankshaft. This is, however, usually not the case with medium-speed engines for marine or power plant applications, [15]. As the rotational speed for these engines is usually in the range 400–1000 rpm, the lowest orders of the tangential torque are close to the first natural frequencies of the rotating shaft system. The implication is that the dynamics of the engine are significantly affected by the rotational behaviour of the rotor/propulsion shaft, which should therefore be taken into account in the engine model. It has been shown that, given a rigid crankshaft, an engine model comprising of two masses and an ideal shaft with a given stiffness and damping represents medium-speed power and marine




doi: 10.1049/iet-cta:20070479

engines sufficiently well for the purpose of balancing the torque contributions [15]. For engine-generator sets where the generator is coupled via a transformer to an infinite bus, the engine model is supplemented with an additional ideal shaft between the generator and infinite bus representing the dynamics between the rotor and bus, Fig. 2.

The engine-generator set depicted in Fig. 2 is described by the equations

$$J_1(\varphi_1)\ddot{\varphi}_1 + \frac{1}{2}\frac{\mathrm{d}J_1(\varphi_1)}{\mathrm{d}\varphi_1}\dot{\varphi}_1^2 + D_1\dot{\varphi}_1 + C_1(\dot{\varphi}_1 - \dot{\varphi}_2) + K_1(\varphi_1 - \varphi_2) = M_u(t) \tag{6}$$

$$J_2\ddot{\varphi}_2 + C_2\dot{\varphi}_2 + K_2\varphi_2 - C_1(\dot{\varphi}_1 - \dot{\varphi}_2) - K_1(\varphi_1 - \varphi_2) = M_{\mathrm{L}}(t) \tag{7}$$

where $J_2$ is the inertia of the load; $K_1$ and $K_2$ the stiffness between the engine and load and the load and grid, respectively; $C_1$ and $C_2$ the viscous damping between engine and the rotor and the rotor and grid, respectively; $D_1$ the friction of the engine; $\ddot{\varphi}_2$, $\dot{\varphi}_2$ and $\varphi_2$ the angular acceleration, speed and position of the load, respectively; and $M_{\mathrm{L}}$ the external load torque affecting the load side of the flexible coupling. Note that the parameters $K_1$ and $C_1$ of the flexible coupling usually depend on the nominal torque level or the vibratory frequency [17], but this dependency has been suppressed in the equations.

In marine diesel-electric installations with multiple engine-generator sets (genset) connected to a common bus, the gensets are dynamically connected to each other. In these installations resonance frequencies and interaction of orders 1/2 and 1 between the gensets generally occur. By using a single angular speed measurement, it is possible for these systems to reconstruct only the total torque consisting of the combustion and external torques. Therefore measurements of the angular speeds $\dot{\varphi}_1$ and $\dot{\varphi}_2$ on both sides of the flexible coupling are required, separating the fuel combustion and external torques from each other, (6) and (7). In contrast, for gensets connected to an infinite grid and with a balanced shaft system, there are normally no external torque excitations at the frequencies considered in cylinder balancing of internal combustion engines. It follows that $\dot{\varphi}_2$ could be estimated based on $\dot{\varphi}_1$ and the dynamics of the flexible coupling.

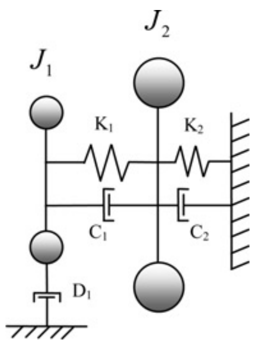


**Figure 2** *Schematic diagram of a medium-speed genset*

## 4 Torque-order model

To solve the cylinder balancing problem described in Section 2, the relation between fuel-injection durations $\mathbf{u}$ and the torque-order components $M_{\mathrm{p}}(\mathbf{u})$ in (1) should be known. In this section, a model between the fuel-injection durations $\mathbf{u}$ and the amplitudes and phases of torque orders $M_{\mathrm{p}}(\mathbf{u})$ is constructed using engine-specific phase angles.

Consider the torque vector $M_{\mathrm{p}}(\mathbf{u})$ depicted in Fig. 3, where the phase $\phi_p(\mathbf{u})$ is defined with respect to the top-dead-centre of cylinder 1. The total torque is composed of cylinder-wise torque contributions

$$M_{\mathrm{p}}(\mathbf{u}) = \sum_{i=1}^{N_{\mathrm{cyl}}} M_{\mathrm{p}}^{(i)}(u_i)$$

where $M_{\mathrm{p}}^{(i)}(u_i) = |M_{\mathrm{p}}^{(i)}(u_i)|\mathrm{e}^{j\alpha_{\mathrm{p}}^{(i)}}$ are complex numbers which define the magnitudes $|M_{\mathrm{p}}^{(i)}(u_i)|$ and phases $\alpha_{\mathrm{p}}^{(i)}$ of the torque contributions from the individual cylinders. The phase angles $\alpha_{\mathrm{p}}^{(i)}$ of the cylinder-wise torque contributions are engine-specific parameters and can be assumed known. For example, in Fig. 4 the phase-angle diagrams for orders 1/2, 1, 3/2 and 3 are depicted for a six-cylinder engine with the firing sequence 1-5-3-6-2-4.

For a diesel engine with a constant efficiency, the magnitude of the torque contribution from an individual cylinder $i$ can be assumed to depend linearly on the fuel-injection adjustment $\Delta u_i$, whereas the shape of the generated torque as a function of crank angle is not affected, [3]. It follows that

$$M_{\mathrm{p}}^{(i)}(u_i + \Delta u_i) = M_{\mathrm{p}}^{(i)}(u_i) + a_{\mathrm{p}}^{(i)}\Delta u_i$$

where

$$a_{\mathrm{p}}^{(i)} = b_p \mathrm{e}^{j\alpha_{\mathrm{p}}^{(i)}} \tag{8}$$

is a complex number aligned with $M_{\mathrm{p}}^{(i)}(u_i)$, whose magnitude $b_p$ denotes the gain from $\Delta u_i$ to the torque-order component in question.

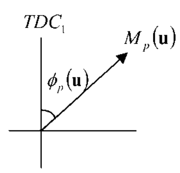


**Figure 3** *Torque vector*


doi: 10.1049/iet-cta:20070479

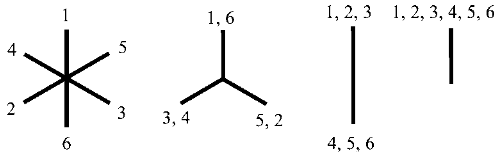


**Figure 4** *Phase-angle diagrams for orders (from left) 1/2, 1, 3/2 and 3 of a six-cylinder engine with symmetric firings*

Introducing the complex-valued vectors

$$\mathbf{a}_{\mathrm{p}} = \begin{bmatrix} a_{\mathrm{p}}^{(1)} \\ \vdots \\ a_{\mathrm{p}}^{(N_{\mathrm{cyl}})} \end{bmatrix}, \quad p \in \mathrm{P}$$

where $\mathbf{P}$ is the set of considered vibration orders; the total order increment associated with a fuel-injection adjustment vector $\mathbf{\Delta u}$ can be written compactly as

$$M_{\mathrm{p}}(\mathbf{u} + \mathbf{\Delta u}) = M_{\mathrm{p}}(\mathbf{u}) + \mathbf{\Delta u}^{\mathrm{T}} \mathbf{a}_{\mathrm{p}} \tag{9}$$

## 5 Cylinder-balancing algorithm

In this section, we present an online cylinder balancing method based on the criterion defined in Section 2, which uses the reconstructed oscillating gas torque $M_u$ and the torque-order model presented in Section 4.

Minimisation of the cost (2) subject to the constraints (3) and (4) is equivalent to minimisation of the Lagrange function

$$h_{\mathrm{L}}(\mathbf{u}) = \frac{1}{2}\sum_{p\in\mathbf{P}} w_p \left|M_{\mathrm{p}}(\mathbf{u})\right|^2 + \frac{1}{2}\sum_{i=1}^{N_{\mathrm{cyl}}} q_i \left|T_i(u_i) - T_{\mathrm{a}}(\mathbf{u})\right|^2 \tag{10}$$

where $q_i/2$ are non-negative Lagrange multipliers associated with the constraint (4).

The objective is to construct an online procedure to solve the constrained optimisation problem defined by (2), (3) and (4), by adjusting the fuel injections according to

$$\mathbf{u}(l+1) = \mathbf{u}(l) + \mathbf{\Delta u}(l) \tag{11}$$

where $l$ is the iteration index. As the total fuel injection $u_{\mathrm{tot}}$ is determined separately by the speed controller, condition (3) implies that the fuel-injection increments calculated by the cylinder-balancing algorithm should satisfy

$$\sum_{i=1}^{N_{\mathrm{cyl}}} \Delta u_i(l) = 0 \tag{12}$$

The fuel-injection adjustment (11) will be determined in such a way that the cost (10) is decreased. For this purpose, consider the cost increment because of the fuel-injection adjustments $\mathbf{u} + \mathbf{\Delta u}$. From (9), we have for the torsional vibration terms

$$\left|M_{\mathrm{p}}(\mathbf{u} + \mathbf{\Delta u})\right|^2 = \mathbf{\Delta u}^{\mathrm{T}} \mathbf{a}_{\mathrm{p}} \left(\mathbf{a}_{\mathrm{p}}^*\right)^{\mathrm{T}} \mathbf{\Delta u} + \mathbf{\Delta u}^{\mathrm{T}} \left(\mathbf{a}_{\mathrm{p}} M_{\mathrm{p}}^*(\mathbf{u}) + \mathbf{a}_{\mathrm{p}}^* M_{\mathrm{p}}(\mathbf{u})\right) + \left|M_{\mathrm{p}}(\mathbf{u})\right|^2 \tag{13}$$

where $\mathbf{a}_{\mathrm{p}}^*$ denotes the complex conjugate.

In a similar way, a first-order expansion of the cylinder-wise exhaust-gas temperatures gives

$$T_i(u_i + \Delta u_i) = T_i(u_i) + \mu \Delta u_i \tag{14}$$

where the gain $\mu > 0$ is assumed equal for all cylinders. Notice that this implies that the average exhaust-gas temperature is not affected if the fuel-injection adjustment satisfies (12), that is, we have

$$T_{\mathrm{a}}(\mathbf{u} + \mathbf{\Delta u}) = T_{\mathrm{a}}(\mathbf{u})$$

It follows that

$$T_i(u_i + \Delta u_i) - T_{\mathrm{a}}(\mathbf{u} + \mathbf{\Delta u}) = T_i(u_i) - T_{\mathrm{a}}(\mathbf{u}) + \mu \Delta u_i$$

and hence

$$\begin{aligned} &\sum_{i=1}^{N_{\mathrm{cyl}}} q_i \left|T_i(u_i + \Delta u_i) - T_{\mathrm{a}}(\mathbf{u} + \mathbf{\Delta u})\right|^2 \\ &= \sum_{i=1}^{N_{\mathrm{cyl}}} q_i \left|T_i(u_i) - T_{\mathrm{a}}(\mathbf{u})\right|^2 + \mu^2 \mathbf{\Delta u}^{\mathrm{T}} \mathbf{Q}\, \mathbf{\Delta u} \\ &\quad + 2\mu \tilde{\mathbf{T}}(\mathbf{u})^{\mathrm{T}} \mathbf{Q}\, \mathbf{\Delta u} \end{aligned} \tag{15}$$

where $\mathbf{Q} = \mathrm{diag}\left(q_1, q_2, \ldots, q_{N_{\mathrm{cyl}}}\right)$ and

$$\tilde{\mathbf{T}}(\mathbf{u}) = \begin{bmatrix} T_1(u_1) - T_{\mathrm{a}}(\mathbf{u}) \\ T_2(u_2) - T_{\mathrm{a}}(\mathbf{u}) \\ \vdots \\ T_{N_{\mathrm{cyl}}}\left(u_{N_{\mathrm{cyl}}}\right) - T_{\mathrm{a}}(\mathbf{u}) \end{bmatrix}$$

Combining (10), (13) and (15) gives

$$h_{\mathrm{L}}(\mathbf{u} + \mathbf{\Delta u}) = h_{\mathrm{L}}(\mathbf{u}) + \mathbf{\Delta u}^{\mathrm{T}} \mathbf{V}\, \mathbf{\Delta u} + \mathbf{g}(\mathbf{u})^{\mathrm{T}} \mathbf{\Delta u} \tag{16}$$

where

$$\mathbf{V} = \sum_{p\in\mathrm{P}} w_p \mathbf{a}_{\mathrm{p}} \left(\mathbf{a}_{\mathrm{p}}^*\right)^{\mathrm{T}} + \mu^2 \mathbf{Q}$$



doi: 10.1049/iet-cta:20070479

and

$$\mathbf{g}(\mathbf{u}) = \sum_{p\in P} w_p \left( M_p^*(\mathbf{u})\mathbf{a}_p + M_p(\mathbf{u})\mathbf{a}_p^* \right) + 2\mu \mathbf{Q}\tilde{\mathbf{T}}(\mathbf{u}) \quad (17)$$

The Newton method for fuel-injection adjustment $\Delta\mathbf{u}$ corresponds to minimising the quadratic function (16) subject to the linear equality constraint (12). The constraint can be conveniently incorporated by the reduced gradient method as follows [18]. Introduce an $N_{cyl}$-dimensional vector $\mathbf{s}$ and an $N_{cyl} \times (N_{cyl} - 1)$ matrix $\mathbf{Z}$ such that $[\mathbf{s} \quad \mathbf{Z}]$ is non-singular and

$$[1 \quad 1 \quad \ldots \quad 1][\mathbf{s} \quad \mathbf{Z}] = [1 \quad 0 \quad 0 \quad \ldots \quad 0]$$

Then, any $\Delta\mathbf{u}(l)$ can be decomposed as $\Delta\mathbf{u}(l) = \mathbf{s}\,\Delta u_s(l) + \mathbf{Z}\,\Delta\mathbf{u}_r(l)$, and in particular, any $\Delta\mathbf{u}(l)$ that satisfies (3) is given by

$$\Delta\mathbf{u}(l) = \mathbf{Z}\,\Delta\mathbf{u}_r(l) \quad (18)$$

where $\Delta\mathbf{u}_r(l)$ is a vector with dimension $N_{cyl} - 1$.

Introducing (18) into (16) gives

$$h_L(\mathbf{u} + \mathbf{Z}\,\Delta\mathbf{u}_r) = h_L(\mathbf{u}) + \Delta\mathbf{u}_r^T\mathbf{Z}^T\mathbf{V}\mathbf{Z}\,\Delta\mathbf{u}_r + \mathbf{g}(\mathbf{u})^T\mathbf{Z}\,\Delta\mathbf{u}_r \quad (19)$$

The Newton method for fuel-injection adjustment takes the form

$$\mathbf{u}(l+1) = \mathbf{u}(l) - \kappa\mathbf{Z}\left(\mathbf{Z}^T\mathbf{V}\mathbf{Z}\right)^{-1}\mathbf{Z}^T\mathbf{g}(\mathbf{u}(l)) \quad (20)$$

where $\kappa$ is a non-negative step length.

The weights $q_i$ in (10) may be considered as design parameters to be selected in such a way that the constraints (4) on the temperature deviations are achieved. An alternative is to adjust the weights adaptively to satisfy the Kuhn–Tucker conditions associated with the constraints (4)

$$q_i\left(\left|T_i(u_i) - T_a(\mathbf{u})\right|^2 - \Delta T_{max}^2\right) = 0, \quad i = 1, 2, \ldots, N_{cyl} \quad (21)$$

This can be achieved by adjusting the weights according to the Robbins–Monro scheme [19, 20]

$$q_i(l+1) = q_i(l) + \gamma q_i\left(\left|\tilde{T}_i(\mathbf{u}(l))\right|^2 - \Delta T_{max}^2\right) = 0, \quad i = 1, 2, \ldots, N_{cyl} \quad (22)$$

where $\gamma$ is a positive step length parameter. A similar procedure has been used in [21] to recursively adjust the Lagrange multiplier of a constrained control problem on line.

An iterative cylinder-balancing algorithm can now be constructed which uses the reconstructed gas torque $M_u$ and the measured exhaust-gas temperatures to adjust the cylinder-wise fuel-injection durations in such a way that the torsional vibrations of the crankshaft are minimised in an optimal way.

*Algorithm:*
Stage 0. Specify parameters $w_p$, $p \in \mathbf{P}$, $\Delta T_{max}$, $q_i$, $\kappa$, $\gamma$, sampling time $T$ for speed measurements and time $T_f$ between fuel adjustments. Set $l = 0$.

Stage 1. Measure the angular speeds of the flywheel $\dot{\varphi}_1(kT)$ and the load $\dot{\varphi}_2(kT)$ at sampling instants $kT$ in time interval $lT_f < kT \leq (l+1)T_f$, and the cylinder-wise exhaust-gas temperatures $T_i(u_i)$ during the time period.

Stage 2. Calculate the superposed oscillating torque $M_u$ using (6) and determine the Fourier series coefficients $M_p(\mathbf{u})$ of $M_u$ in (1).

Stage 3. Adjust the cylinder-wise fuel injections according to (20) and the weights $q_i$ according to (22) if required.

Stage 4. Set $l := l + 1$ and continue from stage 1.

Notice that the calculations in stage 2 can be done efficiently by a frequency-domain representation of (6), [15].

## 6 Experimental results

The proposed method was evaluated on a 6 MW genset, with a turbocharged six-cylinder Wärtsilä 6L46CR common-rail diesel engine. The engine was connected to a synchronous generator producing electricity to an infinite bus. The standard engine speed pickup was used for measuring the angular speed of the flywheel. The angular speed of the generator rotor was measured by means of an AVL encoder, providing 360 pulses per revolution. These pulse trains were converted to voltage signals proportional to the angular deflection of each mass. The signals were sampled at the frequency 6 kHz using a PC and DASYLab™ [22]. The measurements were subsequently analysed by a Matlab™ script and the control adjustment was fed to the engine control system. Prior to each test, the fuel injector calibrations were set to zero.

The cylinder-balancing algorithm described in Section 5 was applied to reduce the torsional vibration orders $p = 1/2$, 1 and 3/2. The phase angles of the cylinder-wise torque contributions are given in Fig. 4, and the values of the order-wise torque amplitude gains in (8) were experimentally determined at $b_{1/2} = 34$, $b_1 = 47$ and $b_{3/2} = 47$ Nm/μs. The exhaust-gas temperature gain in (14) was taken as $\mu = 0.006$°C/μs.

The parameter values used in the algorithm are summarised in Table 1. In the tests presented here the exhaust-gas temperature deviations were constrained by

**Table 1** Parameters used in test

| Parameter name | Parameter | Value |
|---|---|---|
| set of minimised vibration orders | $\mathbf{P}$ | {1/2, 1, 3/2} |
| order weights | $\mathbf{w}_p$ | {1, 0.5, 0.2} |
| exhaust-gas temperature weights | $q_i$ | 2 |
| step length parameter, (20) | $\kappa$ | 0.1 |
| step length parameter, (22) | $\gamma$ | 0 |

using fixed temperature weights $q_i$ in the cost function (10), and no explicit bound on the deviations was specified.

Fig. 5 shows the vibration-order amplitudes, exhaust-gas temperatures and the fuel-injection duration adjustments of an engine test at 10% engine load. It is seen that in only six iterations the torsional vibration amplitudes were reduced by 75% for the orders 1/2 and 1 and by 30% for the order 3/2, and the cylinder-wise temperature deviations were also reduced. The propagation of the torsional vibration amplitudes for an engine test at 50% engine load are shown in Fig. 6, along with the exhaust-gas temperatures and fuel-injection duration adjustments. In this case, the achieved order-wise amplitude reductions were 85, 50 and 20%. Results obtained in an engine test at 100% engine load are shown in Fig. 7. Here, the order amplitudes are reduced by 72, 92 and 10%.

Figs. 5–7 show the converged minimum torsional vibration amplitude levels which were achievable with the cylinder-balancing method, although the fuel injections, and hence the cylinder-wise temperatures, may not have reached their steady-state values. Comparing the order-wise convergence rates, it is seen that the amplitudes for the order 3/2 converge slower than for the orders 1/2 and 1. This can be attributed to the lower weighting used for the order 3/2 in the algorithm, as well as to model errors because of unaccounted crankshaft dynamics. The cylinder-wise temperature deviations are reduced in the tests at 10 and

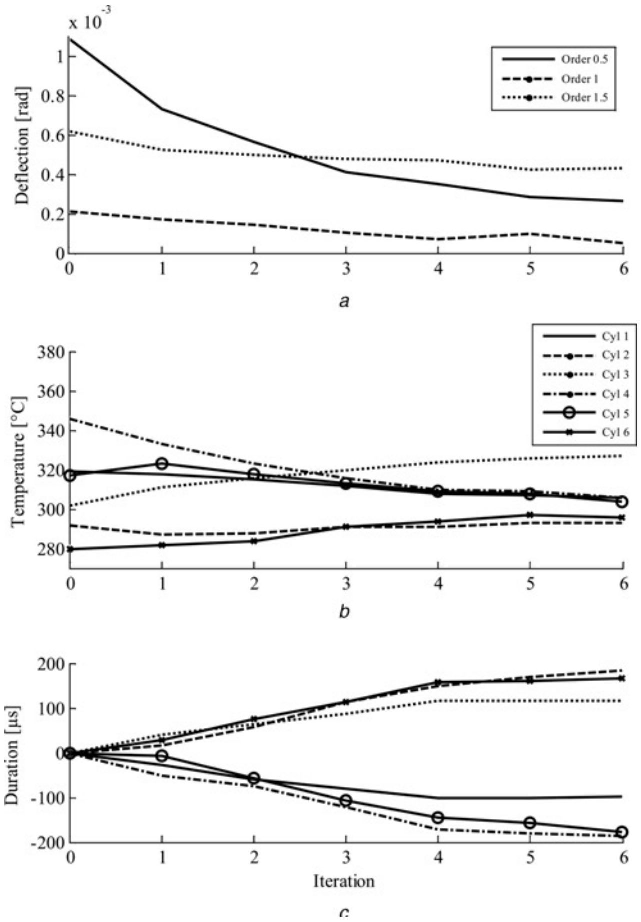


**Figure 5** *Engine test at 10% load*

*a* Propagation of the three first torsional vibration orders
*b* Exhaust-gas temperatures
*c* Fuel-injection durations

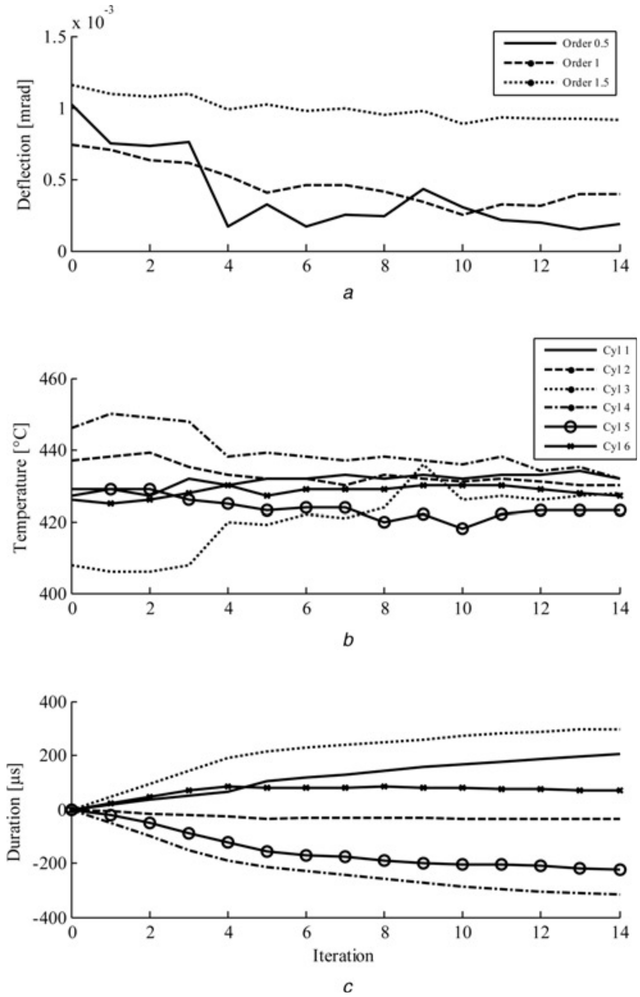


**Figure 6** *Engine test at 50% load*

*a* Propagation of the three first torsional vibration orders
*b* Exhaust-gas temperatures
*c* Fuel-injection durations



*IET Control Theory Appl.*, 2008, Vol. 2, No. 11, pp. 1024–1032
doi: 10.1049/iet-cta:20070479

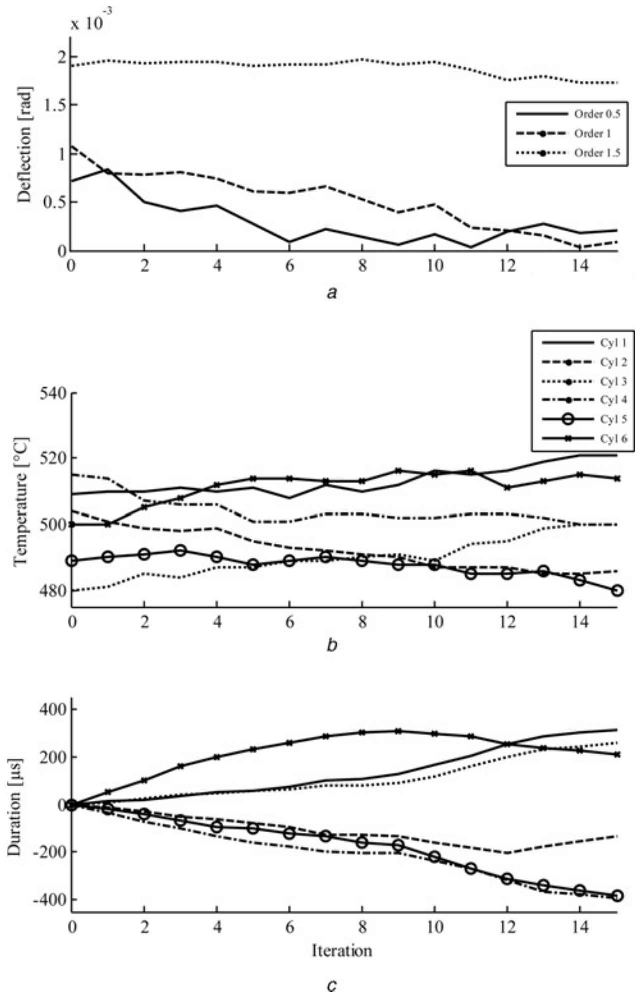


**Figure 7** *Engine test at 100% load*

*a* Propagation of the three first torsional vibration orders
*b* Exhaust-gas temperatures
*c* Fuel-injection durations

50% engine load, whereas a slight increase is obtained at 100% engine load. This could, however, be remedied by increasing the temperature weighting Notice also that the exhaust-gas temperatures in Fig. 7 are grouped in compliance with the phase-angle diagram of order 1, Fig. 4. This behaviour can be traced to the presence of additional crankshaft vibrations of order 1, which are caused by mass imbalances and misalignments in the shafts. It follows that when the torsional vibrations are minimised, the fuel-injection adjustments, and hence the exhaust-gas temperatures, will be aligned in accordance with the dominating vibration order $p = 1$. This may also be a cause for the diverging temperature deviations seen in Fig. 7.

# 7 Conclusions

In this paper, a cylinder-balancing method has been developed which adjusts the fuel-injection durations in such a way that the torsional vibrations of the crankshaft are reduced. In addition, constraints on the cylinder-wise exhaust-gas temperature deviations are taken into account. The vibration reduction is addressed as a constrained online optimisation problem, where the Newton direction of fuel-injection durations are determined based on a lumped-mass model of the engine. The proposed procedure is verified in full-scale engine tests, where the torsional vibrations of a 6 MW Wärtsilä W6L46CR engine were reduced well below admissible levels.

# 8 Acknowledgments

This research was financially supported by Wärtsilä Finland. The authors wish to thank Gösta Liljenfeldt, Johan Pensar and Peter Sundström for their support and numerous helpful discussions, and the personnel at the Wärtsilä engine laboratory in Vaasa, in particular Tapio Malmimäki and Per Lönnqvist, for their assistance and support during the test phase.

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