The signomial global optimization algorithm

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- As the PLFs are updated, the approximations are improved.

The considered class of MINLP problems

The MISP problem formulation

$$
\begin{aligned}
&\text{minimize} && f(\mathbf{x}) && \mathbf{x} = (x_1, x_2, \ldots, x_I) \\
&\text{subject to} && \mathbf{A}\mathbf{x} = \mathbf{a} \quad \mathbf{B}\mathbf{x} \leq \mathbf{b} && \\
& && g_n(\mathbf{x}) \leq 0 && n = 1, 2, \ldots, J_n \\
& && q_m(\mathbf{x}) + \sigma_m(\mathbf{x}) \leq 0 && m = 1, 2, \ldots, J_m
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- The vector $\mathbf{x}$ can contain both continuous and integer-valued variables.
- The differentiable real functions f and g are (pseudo)convex, and the functions q and σ are convex and signomial respectively.

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- A signomial function is a sum of signomial terms, where each term consists of products of power functions, *i.e.,*

$$\sigma(\mathbf{x}) = \sum_{j=1}^{J} c_j \prod_{i=1}^{I} x_i^{p_{ji}}, \qquad c_j,\ p_{ji} \in \mathbb{R}.$$

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Example

$$q(x_1,x_2) + \sigma(x_1,x_2) = \underbrace{x_1^2 + e^{x_2}}_{q(x_1,x_2)} + \underbrace{2.3x_1^{0.35} - 4x_1x_2^{0.5} + x_1x_2}_{\sigma(x_1,x_2)}.$$

The transformation approach

Convexification and relaxation of MISP problems

1. Convexification

- Every signomial term can be transformed to convex form by using single-variable transformations.
- Additional variables and a number of nonlinear equality constraints defining the inverse transformations are obtained.

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- Additional variables and a number of nonlinear equality constraints defining the inverse transformations are obtained.

2. Underestimation, convexification and relaxation

- By using properly selected transformations, the convexified terms are underestimated when the inverse transformations are approximated by piecewise linear functions.
- The MINLP problem is now convexified and relaxed.

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Positive term

One of the following is true:

- All powers are negative

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- One power is positive, the rest negative, and the sum of the powers is ≥ 1

  Example: $x_1^2 x_2^{-1}$

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Negative term

- All powers are positive and the sum of the powers is ≤ 1

  Example: $-x_1^{0.5} x_2^{0.5}$

Convexifying and underestimating signomial terms

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Positive term

For all $i, j: \; p_{ji} > 0$:

- Exponential transformation (ET)
  $x_i = e^{X_{ji}}$
- Positive power transformation (PPT)
  $x_i = {X_{ji}}^{Q_{ji}}$

  $$\begin{cases} Q_{ji} \geq 1 & \text{if } i = k, \\ Q_{ji} < 0 & \text{if } i \neq k, \end{cases} \qquad \sum_{i=1}^{I} p_{ji} Q_{ji} \geq 1.$$
- Negative power transformation (NPT)
  $x_i = {X_{ji}}^{Q_{ji}}, \; Q_{ji} < 0.$

Convexifying and underestimating signomial terms

- A nonconvex signomial term $c_j \prod_{i=1}^{I} x_i^{p_{ji}}$ is convexified by applying certain transformations to the individual variables in the term:

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Negative term

- Power transformation (PT)
  $x_i = X_{ji}{}^{Q_{ji}}$

$$\begin{cases} 0 < Q_{ji} \leq 1 & \text{if } p_{ji} > 0, \\ Q_{ji} < 0 & \text{if } p_{ji} < 0, \end{cases}$$

$$\sum_{i=1}^{I} p_{ji} Q_{ji} \leq 1.$$

- The convexified term is underestimated by replacing the inverse transformations ($X_{ji} = \ln x_i$ or $X_{ji} = x_i^{1/Q_{ji}}$) with PLFs.

Examples of the transformations

Transforming a positive signomial term

Original term	Transformation	Transformed term
	PPT	$X_1^{4.1} X_2^{-0.1} x_3^{-1} x_4^{-2}$
$x_1^{1.2} x_2^{0.1} x_3^{-1} x_4^{-2}$	NPT	$X_1^{-1.2} X_2^{-0.1} x_3^{-1} x_4^{-2}$
	ET	$e^{1.2x_1} e^{0.1x_2} x_3^{-1} x_4^{-2}$

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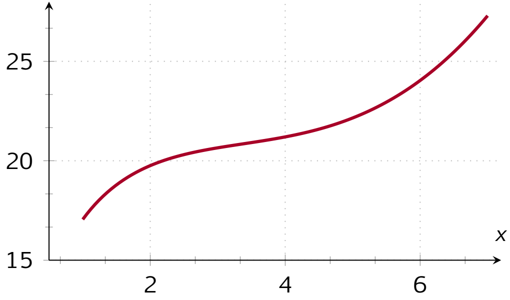
Transforming a negative signomial term

Original term	Transformation	Transformed term
$-x_1^{-1} x_2^{0.5}$	PT	$-X_1^{0.5} x_2^{0.5}$

Example: a univariate nonconvex signomial term

$$f(x) = \underbrace{-8x + 0.05x^3}_{\text{convex}} \underbrace{+25x^{0.5}}_{\text{nonconvex}}, \quad 1 \leq x \leq 7$$



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- A convex underestimator is obtained by applying the ET

$$\hat{f}_1(x, \hat{X}_E) = -8x + 0.05x^3 + 25e^{0.5\hat{X}_E}$$

or one of the PPT or NPT

$$\hat{f}_2(x, \hat{X}_P) = -8x + 0.05x^3 + 25\hat{X}_P^{0.5Q}, \overbrace{Q \geq 2}^{\text{PPT}} \text{ or } \overbrace{Q < 0}^{\text{NPT}}.$$

Here, $\hat{X}_E$ and $\hat{X}_P$ are piecewise linear approximations of the inverse transformations $X_E = \ln x$ and $X_P = x^{1/Q}$.

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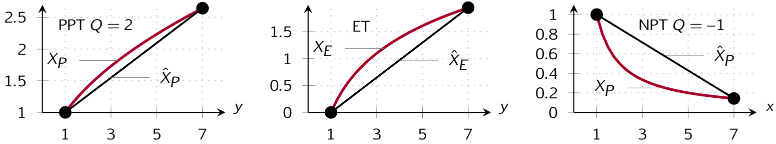

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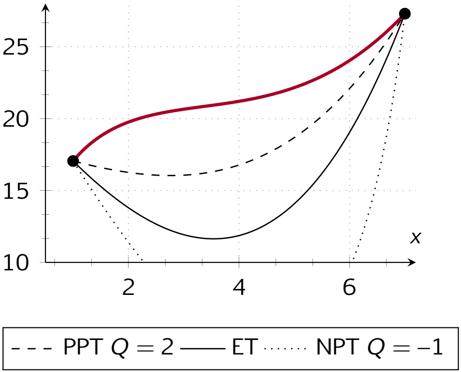

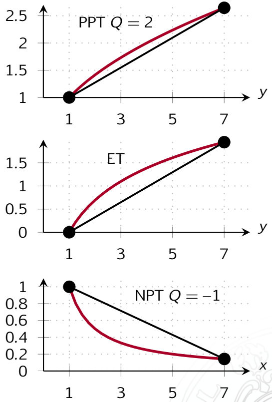

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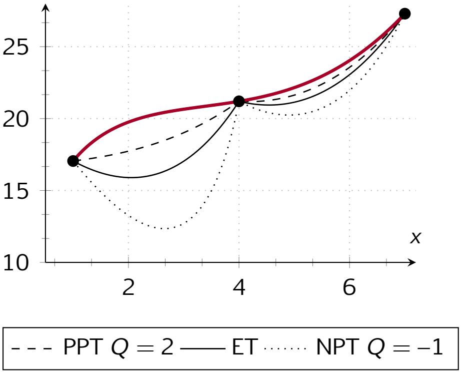

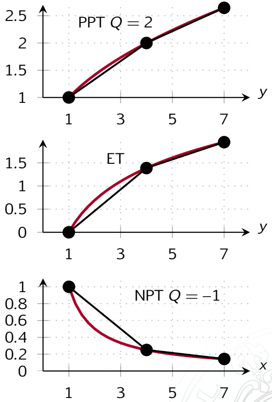

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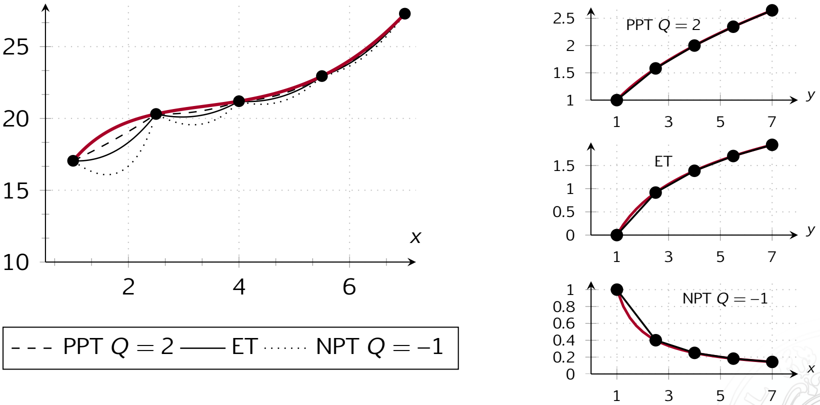
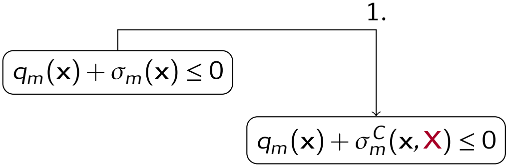


Transformation of the signomial constraints



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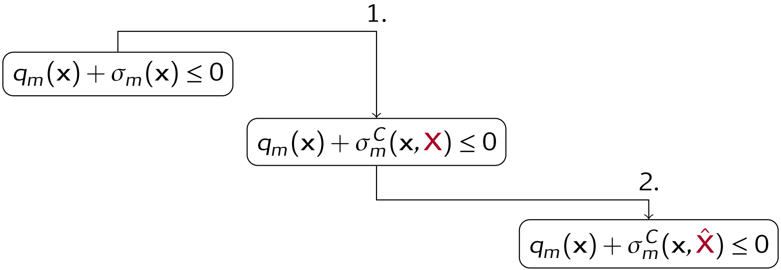


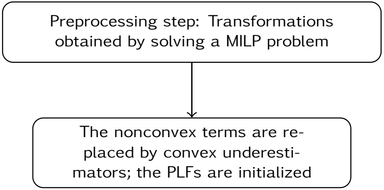
1. **Convexification** of σ_m by transformations $x_i = T_{ji}(X_{ji})$. Nonconvexities moved to $X_{ji} = T_{ji}^{-1}(x_i)$.
2. **Underestimation** of σ_m^C by approximating $X_{ji} = T_{ji}{}^{-1}(x_i)$ with PLFs $\hat{X}_{ji}$. The integer-relaxed problem is now convex and overestimates the original problem.

The signomial global optimization algorithm

Flowchart of the SGO-algorithm

Preprocessing step: Transformations obtained by solving a MILP problem

Flowchart of the SGO-algorithm



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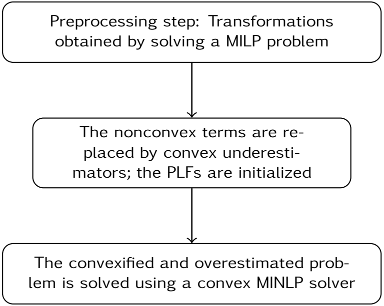

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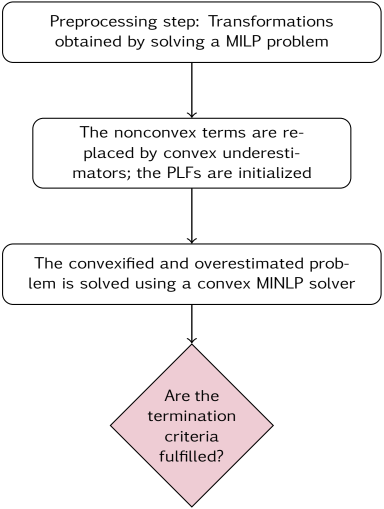

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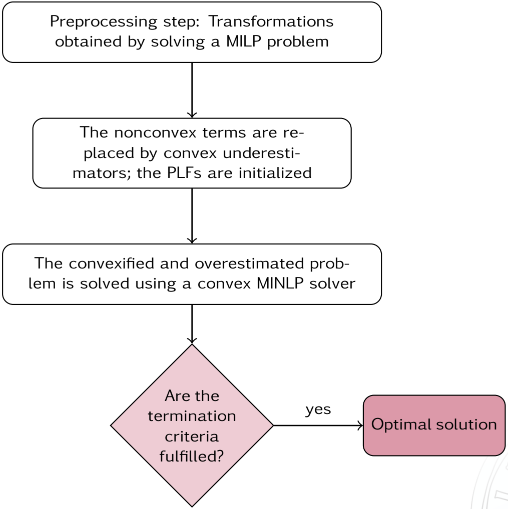

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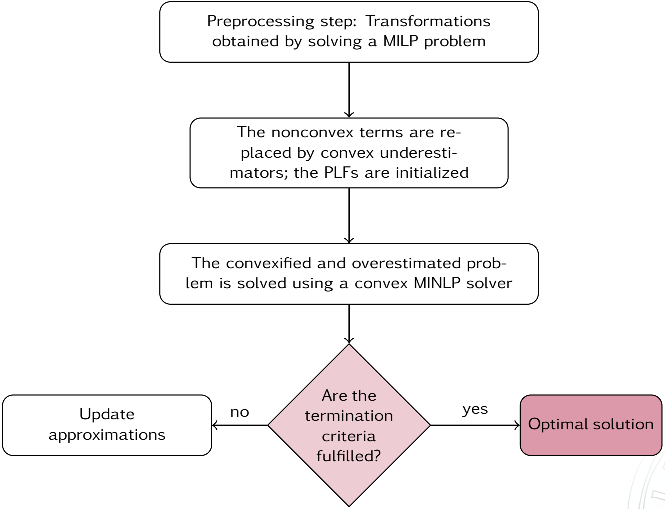

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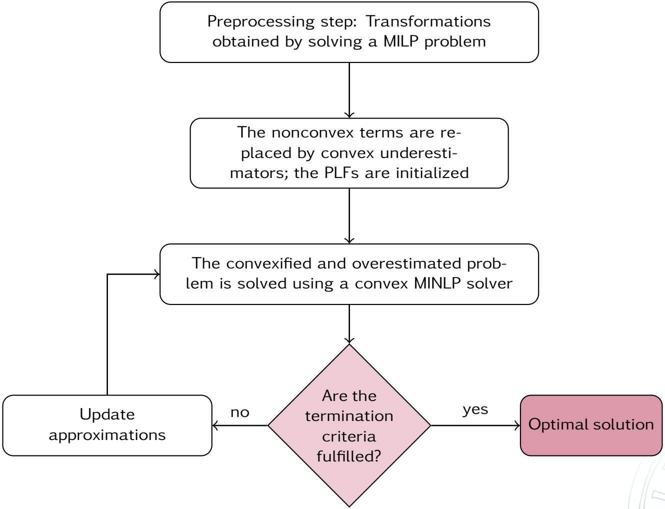

Obtaining an optimized set of transformations

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Solution approach

Create a MILP problem formulation, whose solution determines the optimal set of transformations required to transform the signomial terms, specified by the coefficients c_j and powers p_{ji}.

The objective function of the MILP problem

For a MISP problem with J_T signomial terms and I variables:

$$\begin{aligned}\text{minimize} \quad & \delta_R \sum_{i:\, x_i \in R} B_i + \delta_Z \sum_{i:\, x_i \in Z} B_i + \delta_I \sum_{i=1}^{I} \sum_{j_1=1}^{J_T} \sum_{\substack{j_2=1 \\ j_2 \neq j_1}}^{J_T} \gamma_{j_1 j_2 i} \\ & + \sum_{i=1}^{I} \sum_{j=1}^{J_T} \left(\delta_{NT} b_{ji} + \delta_{NS} \Delta_{ji} + \delta_{ET} b_{ji}^{ET} + \delta_{PT} b_{ji}^{PT} + \delta_P \beta_{ji} \right)\end{aligned}$$

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$$+ \sum_{i=1}^{I} \sum_{j=1}^{J_T} (\delta_{NT} b_{ji} + \delta_{NS} \Delta_{ji} + \delta_{ET} b_{ji}^{ET} + \delta_{PT} b_{ji}^{PT} + \delta_P \beta_{ji})$$

Parameters and variables

- Strategy parameters: $\delta_R, \delta_Z, \delta_I, \delta_{NT}, \delta_{NS}, \delta_{ET}, \delta_{PT}, \delta_P$
- Decision variables (binaries):
  - x_i is transformed in any term $\Rightarrow B_i = 1$
  - x_i is transformed in the j-th term $\Rightarrow b_{ji} = 1$
  - ET or PT used on x_i in the j-th term $\Rightarrow b_{ji}^{ET} = 1 \vee b_{ji}^{PT} = 1$
  - $p_{ji} Q_{ji} > 0 \Rightarrow \beta_{ji} = 1$
  - different transformations for x_i in terms j_1 and $j_2 \Rightarrow \gamma_{j_1 j_2 i} = 1$
- Penalties (real-valued) Δ_{ji} for large/small values of Q_{ji}

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  - favor the PPT or NPT for positive terms

An illustrative example

Two-dimensional example

- The SGO algorithm is now applied to the following MISP problem

$$
\begin{aligned}
\text{minimize} \quad & (x-4)^2 + (y-1)^2 + 2.5x^2y^{-1} \\
\text{subject to} \quad & -20 + 8x - 1.5y^{0.5} + 11y + 0.9y^2 \\
& \quad +0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2 \leq 0 \\
& 1 \leq x \leq 7, \quad 1 \leq y \leq 5, \quad x \in \mathbb{Z}^+, \quad y \in \mathbb{R}^+
\end{aligned}
$$

Two-dimensional example

- The SGO algorithm is now applied to the following MISP problem

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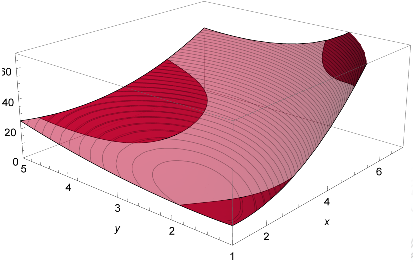

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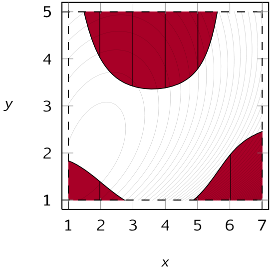

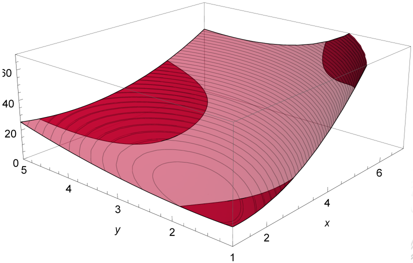

Transformation of the problem

- Optimizing the transformations gives that three transformations are needed

$$\ldots + 0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2 \leq 0.$$

Transformation of the problem

- Optimizing the transformations gives that three transformations are needed

$$\ldots + 0.1x^2\overbrace{y^2 - x^2 + 0.2x^{1.5}y^{0.5}}^{y = Y_1^{-0.5}} - 1.2x\overbrace{y - 2x^{0.5}y^2}^{y = Y_2^{0.25}} \leq 0.$$

Transformation of the problem

- Optimizing the transformations gives that three transformations are needed

$$y = Y_1^{-0.5} \qquad\qquad y = Y_2^{0.25}$$

$$\dots + 0.1x^2 y^2 - x^2 + 0.2x^{1.5} y^{0.5} - 1.2xy - 2x^{0.5} y^2 \leq 0.$$

$$x = X_1^{0.5}$$

Transformation of the problem

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$$\ldots + 0.1x^2\underbrace{\overbrace{y^2 - x^2 + 0.2x^{1.5}y^{0.5}}^{y = Y_1^{-0.5}} - 1.2xy}_{x = X_1^{0.5}} - 2x^{0.5}\overbrace{y^2}^{y = Y_2^{0.25}} \leq 0.$$

- This gives the following convexified constraint

$$\ldots + 0.1x^2 Y_1^{-1} - X_1 + 0.2x^{1.5} Y_1^{-0.25} - 1.2X_1^{0.5} Y_2^{0.25} - 2x^{0.5} Y_2^{0.5} \leq 0.$$

Linear approximation of the transformations

- When the equality constraints

$$X_1 = x^2, \qquad Y_1 = y^{-2}, \qquad \text{and} \qquad Y_2 = y^4$$

are replaced with the PLFs $\hat{X}_1$, $\hat{Y}_1$ and $\hat{Y}_2$, the terms are also underestimated.

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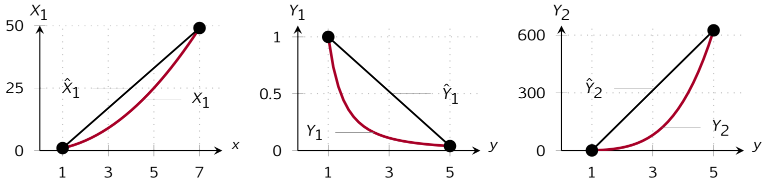

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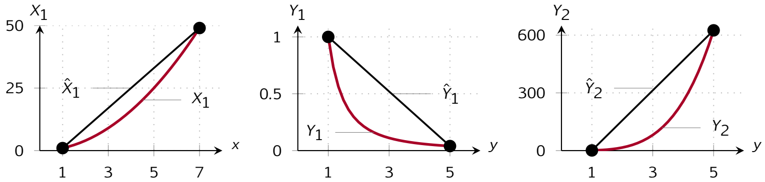
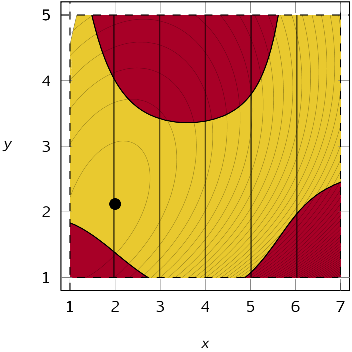
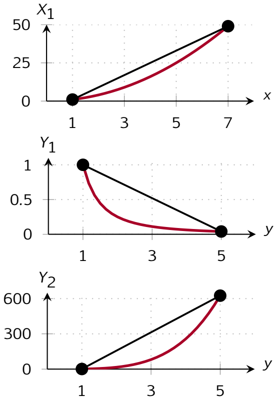


- As new breakpoints are added to the PLFs, the approximations will improve.

Iteration 1



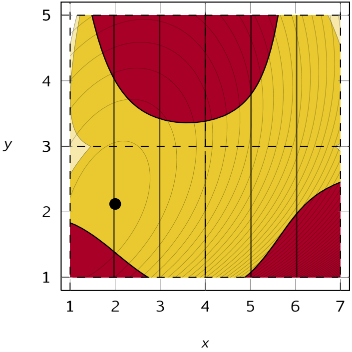


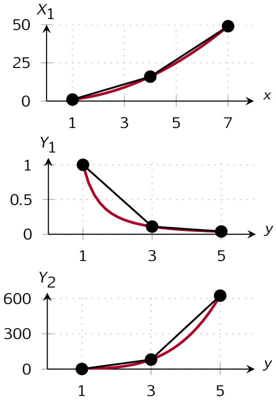
- The value of the objective function is 9.97 for $x = 2$ and $y = 2.12$.
- The original signomial constraint is not fulfilled:

$$\underbrace{\ldots + 0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2}_{= 1.996, \text{ when } x=2,\ y=2.12} \leq 0$$

- Not the global solution to the nonconvex problem!

Iteration 2





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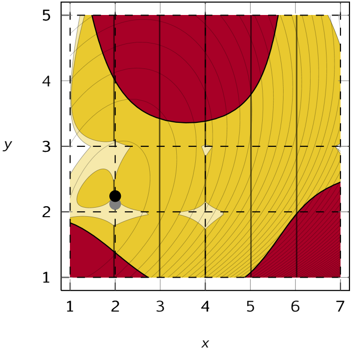
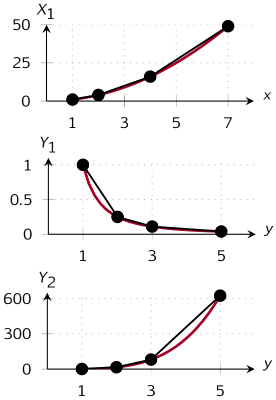
- Not the global solution to the nonconvex problem!

Iteration 3



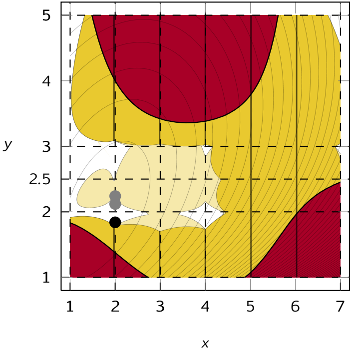


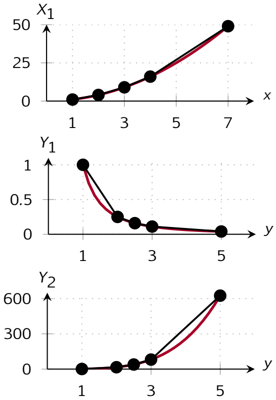
- The value of the objective function is 10.00 for $x = 2$ and $y = 2.24$.
- The original signomial constraint is not fulfilled:

$$\underbrace{\ldots + 0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2}_{= 2.195, \text{ when } x=2,\ y=2.24} \leq 0$$

- Not the global solution to the nonconvex problem!

Iteration 4



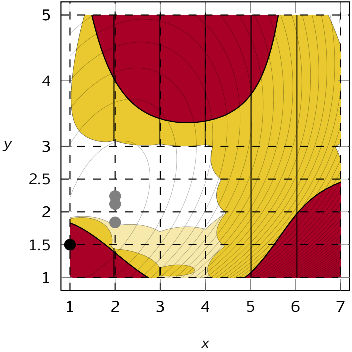


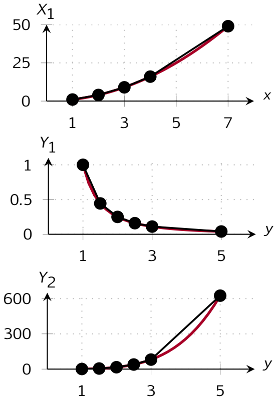
- The value of the objective function is 10.14 for $x = 2$ and $y = 1.84$.
- The original signomial constraint is not fulfilled:

$$\underbrace{\ldots + 0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2}_{= 1.377, \text{ when } x=2,\ y=1.84} \leq 0$$

- Not the global solution to the nonconvex problem!

Iteration 5





- The value of the objective function is 10.92 for $x = 1$ and $y = 1.5$.
- The original signomial constraint is fulfilled:

$$\underbrace{\ldots + 0.1x^2y^2 - x^2 + 0.2x^{1.5}y^{0.5} - 1.2xy - 2x^{0.5}y^2}_{= -2.142,\ \text{when } x=1,\ y=1.50} \leq 0$$

- Globally optimal solution to the nonconvex problem!

Illustration of the convex underestimation

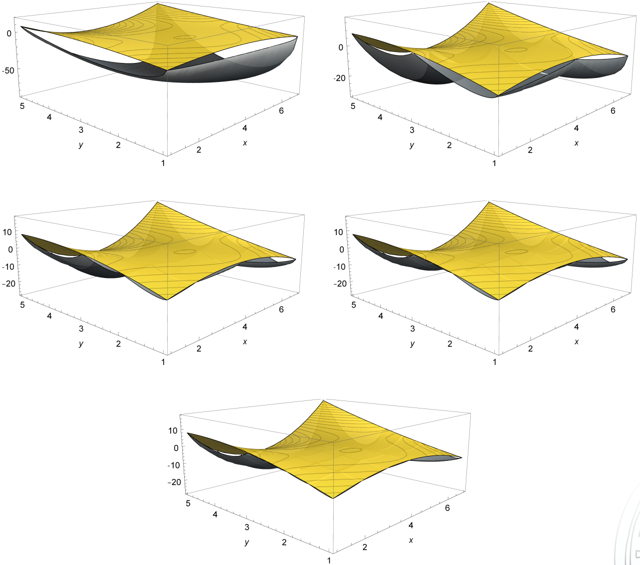
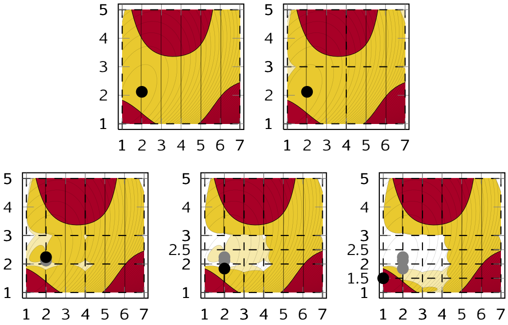


Illustration of the convex underestimation



Iter.	Obj. value	x	y	Constraint
1	9.97	2	2.12	1.996
2	9.97	2	2.12	1.995
3	10.00	2	2.24	2.195
4	10.14	2	1.84	1.377
5	10.92	1	1.50	-2.142

Final remarks

- The SGO-algorithm is a global optimization algorithm for mixed integer signomial programming (MISP) problems.
  - Convex underestimators through single-variable transformations
  - A set of transformations is obtained by solving a MILP problem

Final remarks

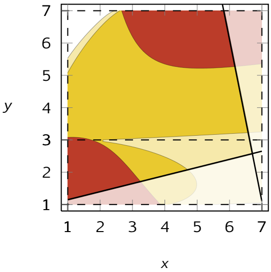
- The SGO-algorithm is a global optimization algorithm for mixed integer signomial programming (MISP) problems.
  - Convex underestimators through single-variable transformations
  - A set of transformations is obtained by solving a MILP problem
- On-going research:
  - Exclusion of unfeasible regions from the search-space.
  - Extension of the current problem-scope to also include any twice-differentiable nonconvex function

Thank you for your attention!

Any questions?

A frequently asked question

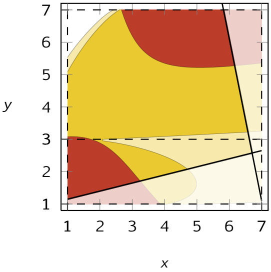
- The following region does not seem to be convex:

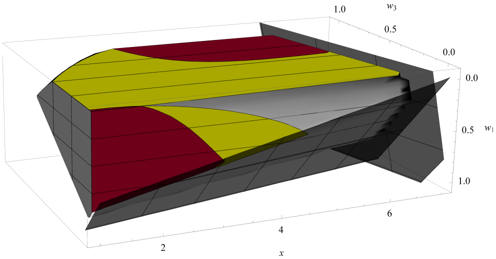


- The relaxed feasible region is convex, however, this is a projection of it for feasible values of the discrete variables defining the PLFs.

A frequently asked question

- The region on the left does not seem to be convex:





- The “projection” (left) and an illustration of the relaxed feasible region (right).