A brief overview of the state/signal approach to infinite-dimensional systems theory

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Optimization and Systems Engineering

Åbo Akademi

Approximate contents

1. What are state/signal systems in discrete time?
2. Some examples of continuous-time systems
3. Well-posed, passive and conservative continuous s/s systems
4. My current research interests (if time permits)
5. Conclusions and references

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What are state/signal systems? [AS1]

The classic state-space model of a discrete-time-invariant system with input u, state x, and output y is

$$\Sigma_{i/s/o}: \quad \begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x(n) \\ u(n) \end{bmatrix}, \quad n \in \mathbb{Z}^+, \tag{1}$$

where $\left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ is bounded on the Hilbert spaces $\left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{U} \end{smallmatrix}\right] \to \left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{Y} \end{smallmatrix}\right]$.

This can be turned into a state/signal system by setting $w(n) := u(n) \dotplus y(n)$ and writing (1) equivalently as

$$\Sigma_{s/s}: \quad \begin{bmatrix} x(n+1) \\ x(n) \\ w(n) \end{bmatrix} \in \left\{ \begin{bmatrix} Ax' + Bu' \\ x' \\ Cx' + Du' + u' \end{bmatrix} \,\middle|\, \begin{bmatrix} x' \\ u' \end{bmatrix} \in \begin{bmatrix} \mathcal{X} \\ \mathcal{U} \end{bmatrix} \right\} =: V, \; n \in \mathbb{Z}^+.$$

Main idea: make minimal distinction between input u and output y.

Useful for unifying i/s/o theory [S1] and systems interconnection!

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What are state/signal systems? [AS1] (cont.)

The subspace $V = \begin{bmatrix} A & B \\ 1 & 0 \\ C & D\dot{+}1 \end{bmatrix} \begin{bmatrix} \mathcal{X} \\ \mathcal{U} \end{bmatrix}$ has the following properties:

1. V is closed in $\begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$, where $\mathcal{W} = \mathcal{U} \dot{+} \mathcal{Y}$
2. If $\begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} \in V$ then $z = 0$
3. The set $\mathcal{D} := \left\{ \begin{bmatrix} x \\ w \end{bmatrix} \mid \begin{bmatrix} z \\ x \\ w \end{bmatrix} \in V \right\}$ is closed in $\begin{bmatrix} \mathcal{X} \\ \mathcal{W} \end{bmatrix}$
4. For every $x \in \mathcal{X}$ there are some x and w such that $\begin{bmatrix} z \\ x \\ w \end{bmatrix} \in V$

Definition

We call any subspace $V \subset \begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$, where $\mathcal{X}, \mathcal{W}$ are Hilbert spaces, with properties (1) – (4) a *discrete-time state/signal system*.

The sequence $\begin{bmatrix} x(n) \\ w(n) \end{bmatrix}$ is a *trajectory* of V if $\begin{bmatrix} x(n+1) \\ x(w) \\ w(n) \end{bmatrix} \in V$, $n \in \mathbb{Z}^+$.

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Input/state/output representations [AS1]

Definition

Let V be a s/s system on $(\mathcal{X},\mathcal{W})$.

A decomposition $\mathcal{W}=\mathcal{U}\dotplus\mathcal{Y}$ is *admissible* for V if there exists a bounded operator $\left[\begin{smallmatrix}A&B\\C&D\end{smallmatrix}\right]:\left[\begin{smallmatrix}\mathcal{X}\\\mathcal{U}\end{smallmatrix}\right]\to\left[\begin{smallmatrix}\mathcal{X}\\\mathcal{Y}\end{smallmatrix}\right]$ s.t. $V=\left[\begin{smallmatrix}A&B\\1&0\\C&D+1\end{smallmatrix}\right]\left[\begin{smallmatrix}\mathcal{X}\\\mathcal{U}\end{smallmatrix}\right]$.

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Semi-canonical i/s/o representations [AS1]

A general s/s system V does not a priori have an i/s/o represent.

However, we can always construct one by choosing the *canonical input space* $\mathcal{U}_0 := \left\{ w \mid \begin{bmatrix} z \\ 0 \\ w \end{bmatrix} \in V \right\}$ and letting the output space $\mathcal{Y}$ be an arbitrary complement to $\mathcal{U}_0$: $\mathcal{W} = \mathcal{U}_0 \dotplus \mathcal{Y}$.
(The output space is not canonical.)

Then $\mathcal{W} = \mathcal{U}_0 \dotplus \mathcal{Y}$ is an admissible i/o decomposition. The corresponding i/s/o representation of V is given by the map

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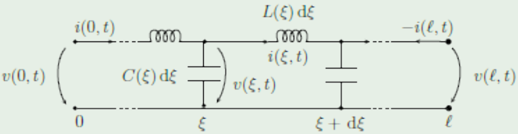
Continuous-time boundary-control examples

Standard example: The transmission line [AKS2]



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$$x(t) = \begin{bmatrix} i(t,\cdot) \\ v(t,\cdot) \end{bmatrix}, \; x(0) \text{ given},$$
$$(t,\xi) \in \mathbb{R}^+ \times [0,\ell].$$

The external signal is $w(t) = (i(t,0), v(t,0), i(t,\ell), v(t,\ell))^\top$.

Much more demanding: n-D spatial domains

The wave equation on a 2-D spatial domain Ω:

$$\frac{\partial^2}{\partial t^2} x(t,\xi,\eta) = c^2 \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) x(t,\xi,\eta), \; (\xi,\eta) \in \Omega.$$

We need Sobolev-space machinery: $\mathcal{W}$ is ∞-dimensional.

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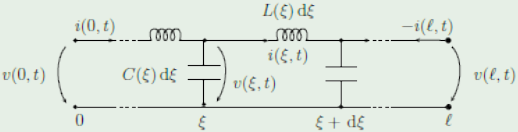

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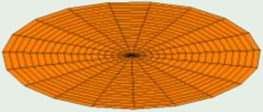

The wave equation on a 2-D spatial domain Ω:

$$\frac{\partial^2}{\partial t^2} x(t,\xi,\eta) = c^2 \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right) x(t,\xi,\eta), \quad (\xi,\eta) \in \Omega.$$

We need Sobolev-space machinery: $\mathcal{W}$ is ∞-dimensional.

Approaching continuous-time s/s systems

In discrete time

$$\begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x(n) \\ u(n) \end{bmatrix}, \quad n \in \mathbb{Z}^+,$$

was turned into

$$\begin{bmatrix} x(n+1) \\ x(n) \\ w(n) \end{bmatrix} \in \begin{bmatrix} A & B \\ 1 & 0 \\ C & D\dot{+}1 \end{bmatrix} \begin{bmatrix} \mathcal{X} \\ \mathcal{U} \end{bmatrix} =: V, \quad n \in \mathbb{Z}^+.$$

In continuous time we have

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A\&B \\ C\&D \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \quad t \in \mathbb{R}^+,$$

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$$\begin{bmatrix} \dot{x}(t) \\ x(t) \\ w(t) \end{bmatrix} \in V, \quad t \in \mathbb{R}^+,$$

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Some problems with continuous time [KS1]

A discrete-time state/signal system $V \subset \begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$ satisfies:

1. V is closed in $\begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$
2. If $\begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} \in V$ then $z = 0$
3. The set $\mathcal{D} := \left\{ \begin{bmatrix} x \\ w \end{bmatrix} \mid \begin{bmatrix} z \\ x \\ w \end{bmatrix} \in V \right\}$ is closed in $\begin{bmatrix} \mathcal{X} \\ \mathcal{W} \end{bmatrix}$
4. For every $x \in \mathcal{X}$ there are some x and w such that $\begin{bmatrix} z \\ x \\ w \end{bmatrix} \in V$

Conditions (i) – (iii) mean that V is the graph of a bounded operator F with domain $\mathcal{D}$: $V = \left\{ \begin{bmatrix} z \\ x \\ w \end{bmatrix} \mid \begin{bmatrix} x \\ w \end{bmatrix} \in \mathcal{D},\ z = F \begin{bmatrix} x \\ w \end{bmatrix} \right\}$.

Continuous time: For all PDE:s F is unbounded!

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Let $V \subset \begin{bmatrix} \mathcal{X} \\ \mathcal{X} \\ \mathcal{W} \end{bmatrix}$ be closed. Then $\left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right] \in \begin{bmatrix} C^1(\mathbb{R}^+;\mathcal{X}) \\ C(\mathbb{R}^+;\mathcal{W}) \end{bmatrix}$ is a *classical trajectory* generated by V if $\begin{bmatrix} \dot{x}(t) \\ x(t) \\ w(t) \end{bmatrix} \in V$ for all $t > 0$.

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Well-posed s/s systems by definition have some admissible i/o decomposition. However, there's no way of constructing it. [KS1]

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A *conservative system* is passive and dissipates no energy.

Mathematically: (For passive systems $\mathcal{W}$ is a Kreĭn space.)

1. The system has "enough" trajectories $\left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right]$.
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"Enough" means more or less that (2) holds for the *dual system*.

Passivity yields surprisingly useful additional structure:
much of the discrete theory can be transferred to continuous time.

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Definition

The *time-domain behaviour* of a discrete s/s system V is the set

$$\mathfrak{W} = \left\{ w \;\middle|\; \left[\begin{smallmatrix} x \\ w \end{smallmatrix}\right] \text{ is a trajectory of } V \text{ with } x(0) = 0 \right\}.$$

The behaviour of a passive s/s system is a passive behaviour:

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Given a passive time-domain behaviour $\mathfrak{W}$, find a passive s/s system V whose time-domain behaviour coincides with $\mathfrak{W}$.

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These articles are very long and technical!

$\Rightarrow$ I am looking for (noncanonical) realisations in the frequency domain using reproducing kernel Hilbert space techniques – hopefully this turns out simpler.

Current research topic: Functional models for s/s systems

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- Main idea is minimal distinction between inputs and outputs.
- Some formulations simplify, but some proofs are unintuitive.
- Discrete-time theory simpler and much more developed.
- Hot topics are realisation theory and interconnection:

I see this as the beginning of a very promising research field.

Thank you for your attention!
Any questions?

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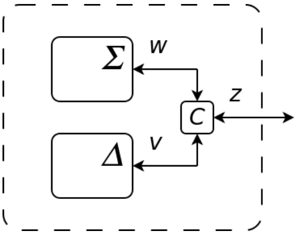

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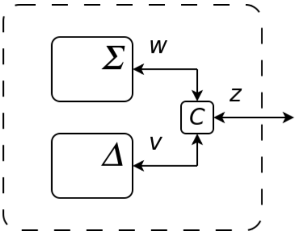

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Selected references

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