OSE SEMINAR 2013

Reformulation of 0-1 Quadratic Programs

Otto Nissfolk

CENTER OF EXCELLENCE IN
OPTIMIZATION AND SYSTEMS ENGINEERING
ÅBO AKADEMI UNIVERSITY

ÅBO, NOVEMBER 15th 2013

Table of contents

- Problem Formulation
  - Coulomb Glass
  - tai*xxc*
  - Boolean Least Squares
  - SDP
- Results
  - CG
  - tai*xxc*
  - BLS

$$E = k\frac{1}{2}\sum_{i}^{N}\sum_{j\neq i}^{N}\frac{x_i x_j}{r_{ij}} + \sum_{i}^{N}\epsilon_i x_i$$

where r_{ij} is the distance between impurities i and j, ϵ_i is the energy for impurity i och x_i is a binary variable stating if impurity i is occupied or not. If n impurities are occupied then $\sum_i^N x_i = n$.

$$E = \frac{1}{2}\mathbf{x}^\mathsf{T}\mathbf{Q}\mathbf{x} + \mathbf{c}^\mathsf{T}\mathbf{x}$$

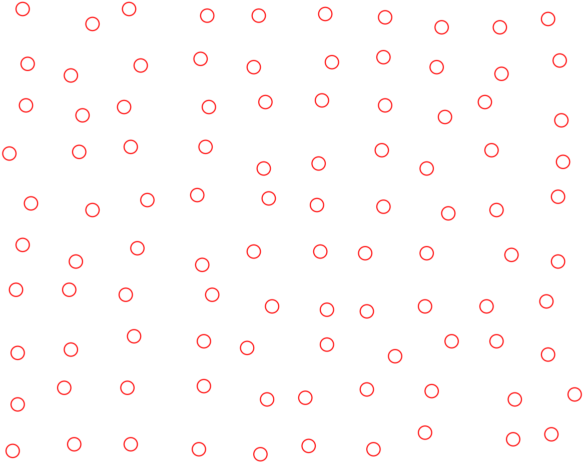
where element $M_{ij} = \frac{1}{r_{ij}}$ and $c_i = \epsilon_i$.

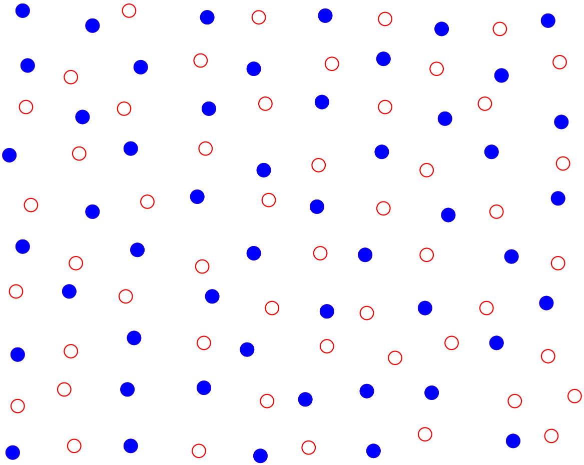
$$\min \mathbf{x}^\mathsf{T} \mathbf{Q} \mathbf{x} + \mathbf{c}^\mathsf{T} \mathbf{x}$$

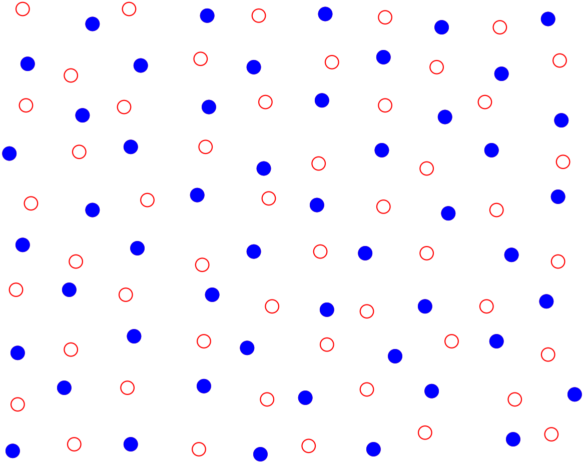
subject to

$$\sum_{i=1}^{n} x_i = \frac{n}{2}$$

$$x_i \in \{0, 1\}$$







$$T_{rstu} = \max_{v,w \in \{-1,0,1\}} \frac{1}{(r-t+nv)^2 + (s-u+nw)^2}$$

$$f_{ij} = \begin{cases} 1 & \text{if } i \leq m \text{ and } j \leq m \\ 0 & \text{otherwise} \end{cases}$$

$$d_{ij} = d_{n(r-1)+s\ n(t-1)+u} = T_{rstu}$$

where (r,s) are the coordinates for i and (t,u) are the coordinates for j

$$\min \mathbf{x}^\mathsf{T}\mathbf{Q}\mathbf{x} + \mathbf{c}^\mathsf{T}\mathbf{x}$$

subject to

$$\sum_{i=1}^{n} x_i = m$$

$$x_i \in \{0, 1\}$$

$$\min \|Ax - b\|^2$$

$$\min \mathbf{x}^\mathbf{T}\mathbf{Q}\mathbf{x} + \mathbf{c}^\mathbf{T}\mathbf{x} + \mathbf{k}$$

where Q is A^TA and A is a random matrix of size $n \times n$, c is $-2A^Tb$ where b is noice and k is b^Tb.

$$\min \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{c}^T \mathbf{x}$$

subject to

$$x_i \in \{0, 1\}$$

$$\begin{aligned} \text{minimize} \quad & \operatorname{tr}(QX) + c^T x \\ \text{subject to} \quad & Ax = a \\ & \operatorname{diag}(X) = x \\ & \begin{bmatrix} 1 & x^T \\ x & X \end{bmatrix} \succeq 0 \end{aligned}$$

$$
\begin{aligned}
\text{minimize} \quad & \operatorname{tr}(QX) + c^T x \\
\text{subject to} \quad & Ax = a \\
& \operatorname{diag}(X) = x \\
& \begin{bmatrix} 1 & x^T \\ x & X \end{bmatrix} \succeq 0 \\
& x_i x_j \geq 0 & \forall i,j \\
& x_i x_j \geq x_i + x_j - 1 & \forall i,j \\
& x_i x_j \leq x_i & \forall i,j \\
& x_i x_j \leq x_j & \forall i,j
\end{aligned}
$$

Problem	UB	LB	Gap	Time	SDP LB	SDP gap	SDP time
50	117.29	116.54	0.62%	2895.8	114.72	2.18%	0.5
100	367.75	367.45	0.08%	9468.3	363.21	1.23%	1.0
150	698.90	694.00	0.70%	14400.6	692.35	0.94%	2.0
200	1097.12	1087.62	0.87%	14400.9	1087.39	0.89%	3.1

Table : Average results for CG problems without strengthened SDP

Problem	UB	LB	Gap	Time	SDP LB	SDP gap	SDP time
50	117.29	116.54	0.62%	2895.8	114.72	2.18%	0.5
100	367.75	367.45	0.08%	9468.3	363.21	1.23%	1.0
150	698.90	694.00	0.70%	14400.6	692.35	0.94%	2.0
200	1097.12	1087.62	0.87%	14400.9	1087.39	0.89%	3.1

Table : Average results for CG problems without strengthened SDP

Problem	UB	LB	Gap	Time	SDP LB	SDP gap	SDP time
50	117.29	117.29	0.00%	2.9	117.27	0.02%	8.1
100	367.75	367.75	0.00%	26.5	367.69	0.02%	8.5
150	698.66	698.66	0.00%	170.8	698.53	0.02%	10.0
200	1096.68	1096.68	0.00%	4621.8	1096.44	0.02%	14.2

Table : Average results for CG problems with strengthened SDP

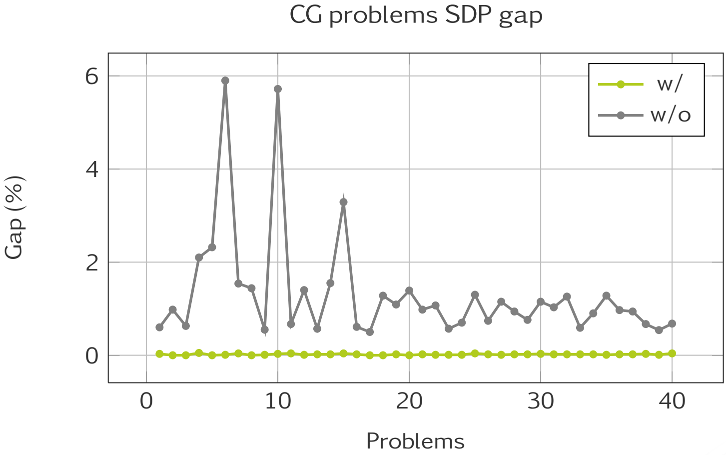


Figure : CG gap

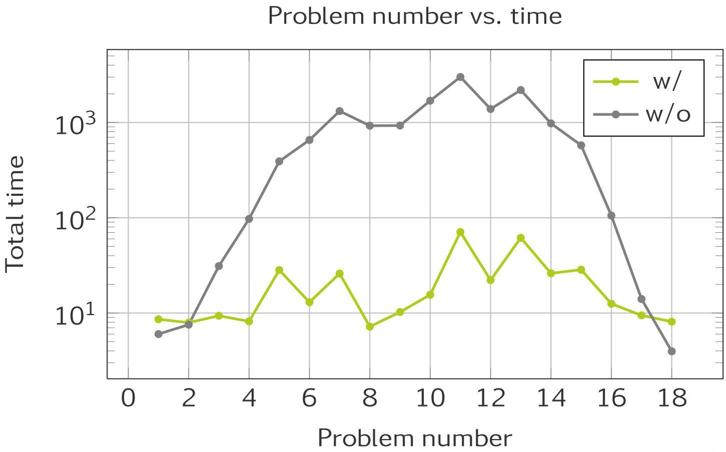


Figure : Solution times for tai36c

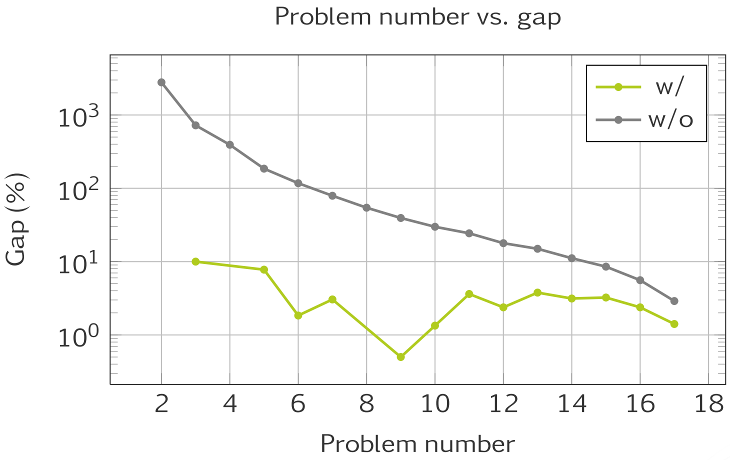
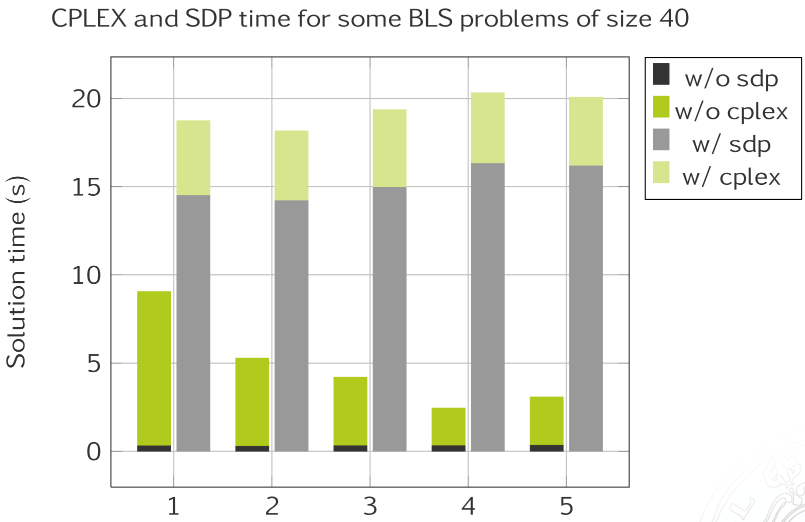
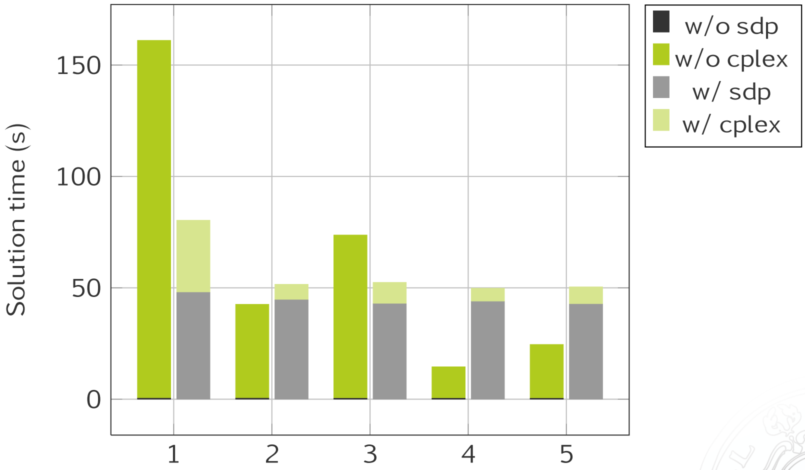


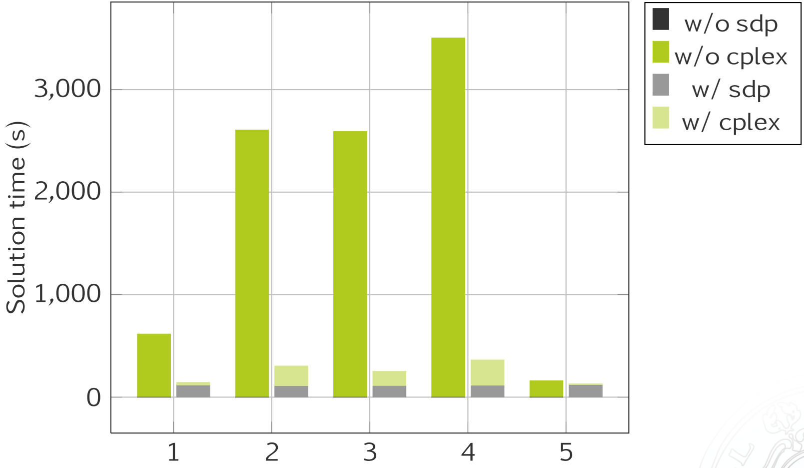
Figure : Gap for tai36c


CPLEX and SDP time for some BLS problems of size 40
Solution time (s)
0
5
10
15
20
1
2
3
4
5
w/o sdp
w/o cplex
w/ sdp
w/ cplex

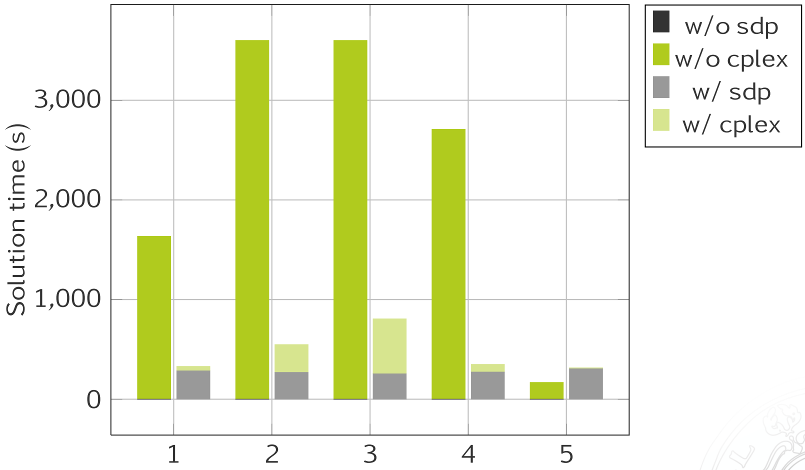
CPLEX and SDP time for some BLS problems of size 60



CPLEX and SDP time for some BLS problems of size 80



CPLEX and SDP time for some BLS problems of size 100



Some references

Alain Billionnet, Sourour Elloumi, and Marie-Christine Plateau.
Improving the performance of standard solvers for quadratic 0-1 programs by a tight convex reformulation: The qcr method.
Discrete Appl. Math., 157:1185–1197, March 2009.

R.E. Burkard, E. Cela, P.M. Pardalos, and L.S. Pitsoulis.
Handbook of Combinatorial Optimization, volume 3.
1998.

C. S. Edwards.
A branch and bound algorithm for the koopmans-beckmann quadratic assignment problem.
Combinatorial Optimization II, 13:35–52, 1980.

Tjalling C. Koopmans and Martin Beckmann.
Assignment problems and the location of economic activities.
Econometrica, 25(1):pp. 53–76, 1957.

É.D. Taillard.
Comparison of iterative searches for the quadratic assignment problem.
Location Science, 3(2):87 – 105, 1995.

Thank you for listening!

Questions?