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Trend and System Identification With Orthogonal Basis Function

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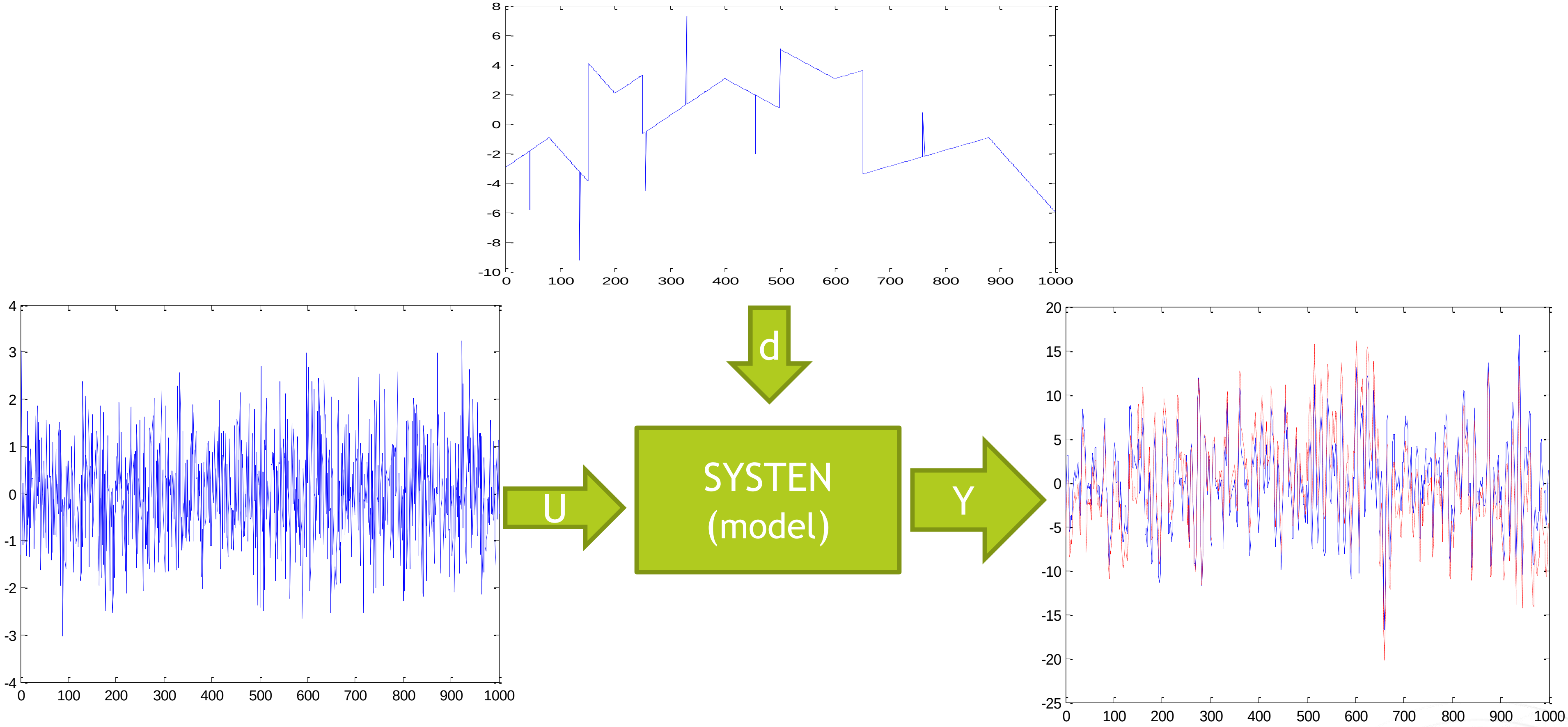
Background

- System identification is difficult when process measurements are corrupted by structured disturbances, such as trends, outliers, level shifts
- Standard approach: removal by data preprocessing but difficult to separate between the effects of known system inputs and unknown disturbances (trends, etc.)
- Orthonormal basis function models are categorized as output-error (ballistic simulation) models

Present contribution

- Identification of system model parameters and disturbances simultaneously
- Sparse optimization used in system identification problem
- Applying the method on simulated and real example

Present contribution



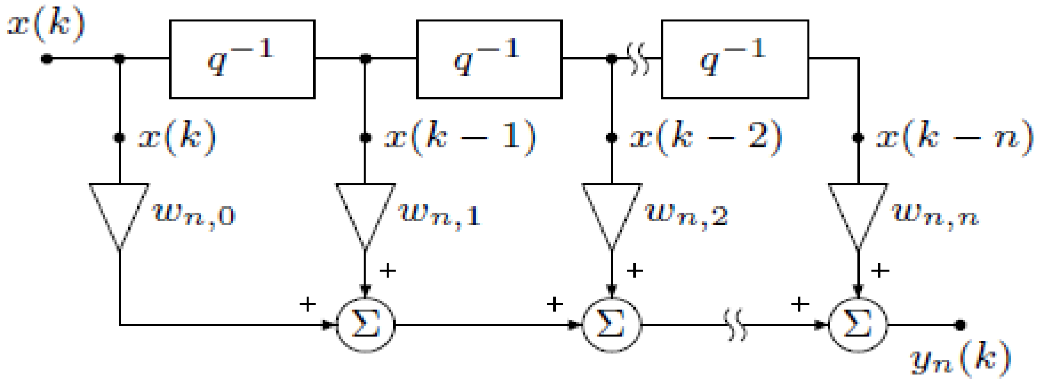
Orthogonal basis function (Fixed-pole model)

Orthogonal basis functions has some advantages:

- The corresponding approximation (representation) has simple and direct solution.
- It corresponds to allpass filters which is robust to implement and use in numerical computation.
- It is popular because a few parameters can describe the system.
- It is a kind of output-error model, and can be insensitive to noise.

Orthogonal basis function (Fixed-pole model)

FIR network:



Laguerre function:

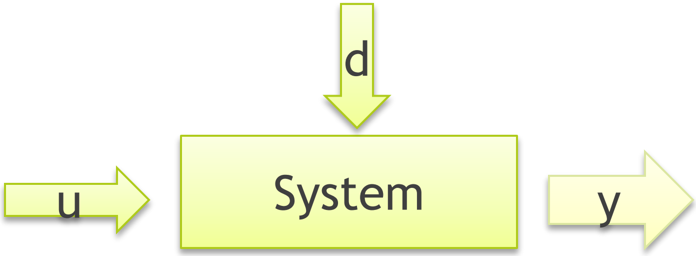
$$L_k(q,\alpha) = \frac{q^{-1}\sqrt{1-\alpha^2}}{1-\alpha q^{-1}} \left(\frac{q^{-1}-\alpha}{1-\alpha q^{-1}} \right)^{k-1}$$

Kautz function:

$$\Psi_{2j}(q,\alpha) = C_2 \frac{q^{-2}}{1+b(c-1)q^{-1}-cq^{-2}} \left(\frac{-c+b(c-1)q^{-1}+q^{-2}}{1+b(c-1)q^{-1}-cq^{-2}} \right)^{j-1}$$

$$j = 1,2,\ldots,n_K/2$$

System Identification



Model: $$y(k) = \varphi(k)^T \theta + d(k)$$

where $$\varphi(k) = [\bar{u}_1(k), \bar{u}_2(k), \ldots, \bar{u}_n(k)]^T, \quad \theta = [a_1, a_2, \ldots, a_n]^T$$

$$\bar{u}_n(k) = L_n(q)u(k) \quad \text{or} \quad \bar{u}_n(k) = \psi_n(q)u(k)$$

The parameters $\hat{\theta}$ can be estimated using standard least squares method:

$$\sum (y(k) - \varphi(k)^T \hat{\theta})^2$$

Problem formulation

We consider the linear system model:

$$y_L(k) = a_1\bar{u}_1(k) + a_2\bar{u}_2(k) + \ldots + a_n\bar{u}_n(k),$$

It is assumed the measured output is given by

$$y(k) = y_L(k) + d(k)$$

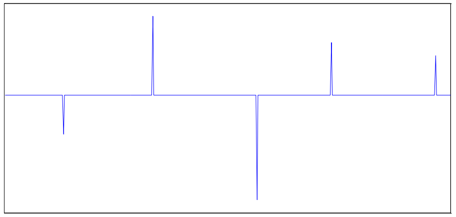
where d(k) is a structured disturbance:

- outlier signal,
- level shifts,
- piecewise constant trends

Disturbance models

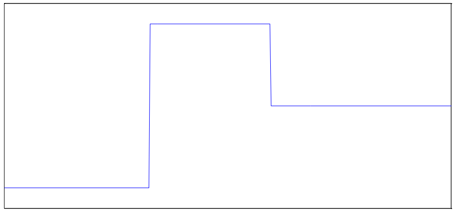
Sequence of outliers:

$$d_0(k) = \begin{cases} d_i, & k = k_i, i = 1, \ldots, M_0 \\ 0, & \text{otherwise} \end{cases}$$



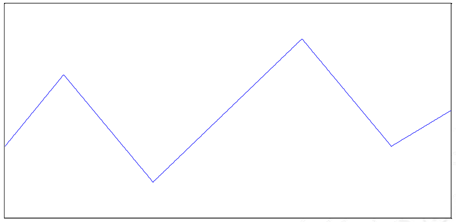
Level shifts:

$$d_1(k) = d_i, \;\; k_i \leq k < k_{i+1}, i = 1, \ldots, M_1$$



Sequence of trends:

$$d_2(k) = d_2(k-1) + \beta_i, k_i \leq k < k_{i+1}, i = 1, \ldots, M_2$$

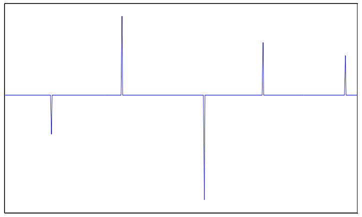


Sparse representation of disturbance

For the structured disturbances, the vectors $D_i d$ are sparse,

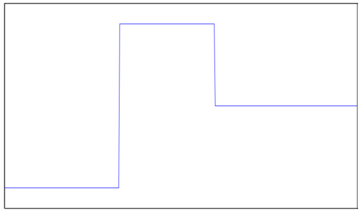
where $d = [d(1) \cdots d(N)]^{\mathrm{T}}$ and , D_i depends on disturbances

Outliers: $D_0 = I$



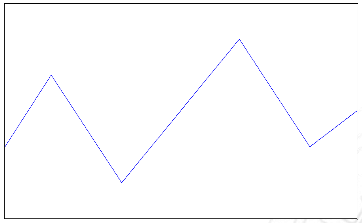
Level shifts:

$$D_1 = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 & 0 \\ 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}$$



Trends:

$$D_2 = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}$$



Sparse optimization approach

Identification by sparse optimization:

$$\min_{\hat{\theta},\hat{d}} \sum_{k} (y(k) - \hat{y}(k))^2$$

subject to $\|D_i d\|_0 \leq M$

where $\|\cdot\|_0 = \text{number of nonzero elements}$

This is an intractable combinatorial optimization problem.

Instead we use $l_1 - \text{relaxation}$ and solve the convex problem

$$\min_{\hat{\theta},\hat{d}} \sum_{k} (y(k) - \hat{y}(k))^2 + \lambda \|D_i \hat{d}\|_1$$

Algorithm

1. Make the basis function expansion based on given prior knowledge of system (pole and system order).

2. Solution of sparse optimization problem by iterative reweighting:
 Minimize the weighted cost to give the estimates

$$\min_{\hat{\theta},\hat{d}} \sum_k (y(k) - \psi^T \hat{\theta} - \hat{d}\,(k))^2 + \lambda \left\| W_i D_i \hat{d} \right\|_1$$

where i = 0, 1 or 2 (user selected).

3. Calculate new weights *W* and go to *step 2*.

$$W = diag(\frac{1}{\varepsilon + |D_i d(k)|})$$

4. Continue until convergence.

5. Use model order reduction to get lower order model.

Example1

We apply the proposed identification detrending method to the ARX model

$$y_0(k) = a_1 y_0(k-1) + a_2 y_0(k-2) + b_1 u(k-1) + b_2 u(k-2) + e(k)$$

$$y(k) = y_0(k) + d(k)$$

with parameter vector

$$\theta = [a_1 \quad a_2 \quad b_1 \quad b_2]^T$$

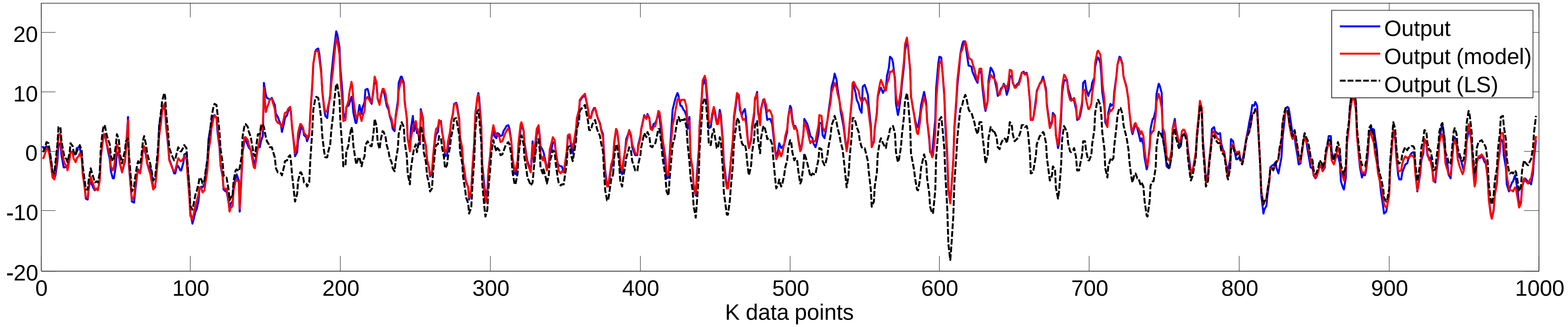
given by

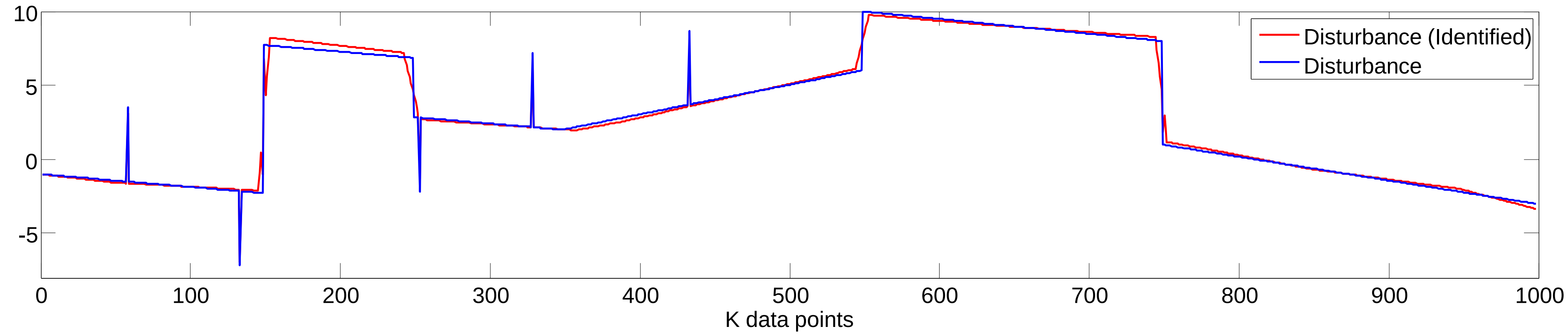
$$\theta = [1.50 \quad -0.7 \quad 1.00 \quad 0.5]^T,$$

u(k) and e(k) are normally distributed signals with variances 1 and 0.1, and d(k) is unknown structured disturbance ,

For making the Kautz basis function we used N=4 as order of system and pole is 0.7 ± 0.4

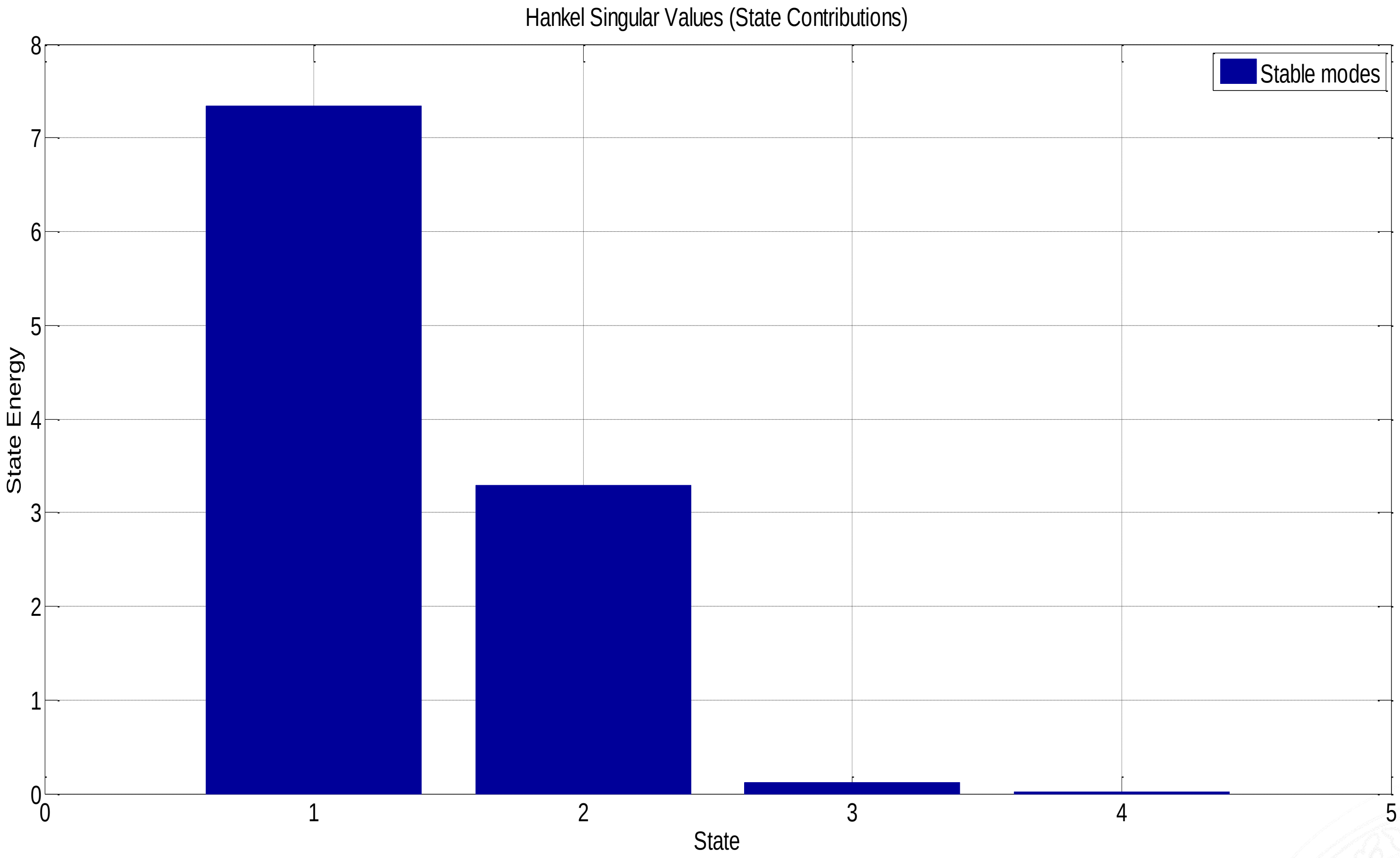
Example1



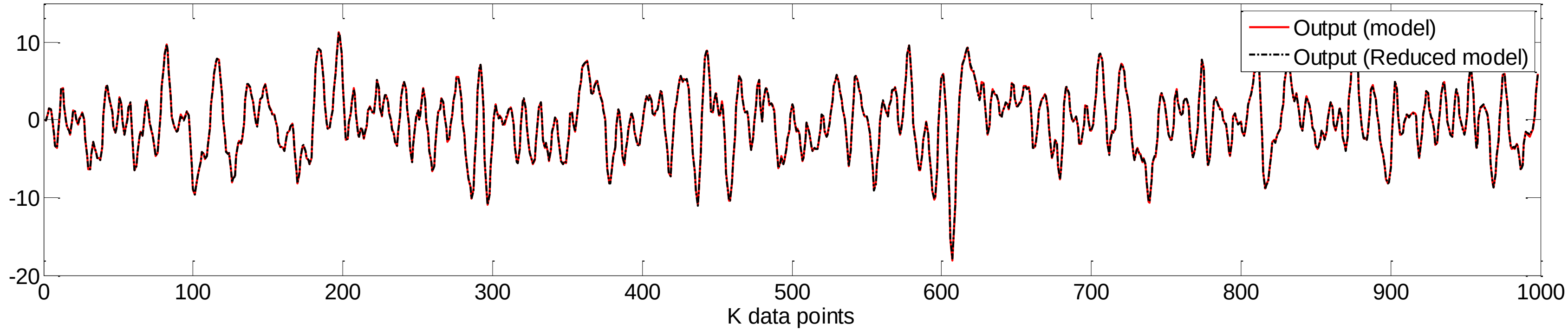


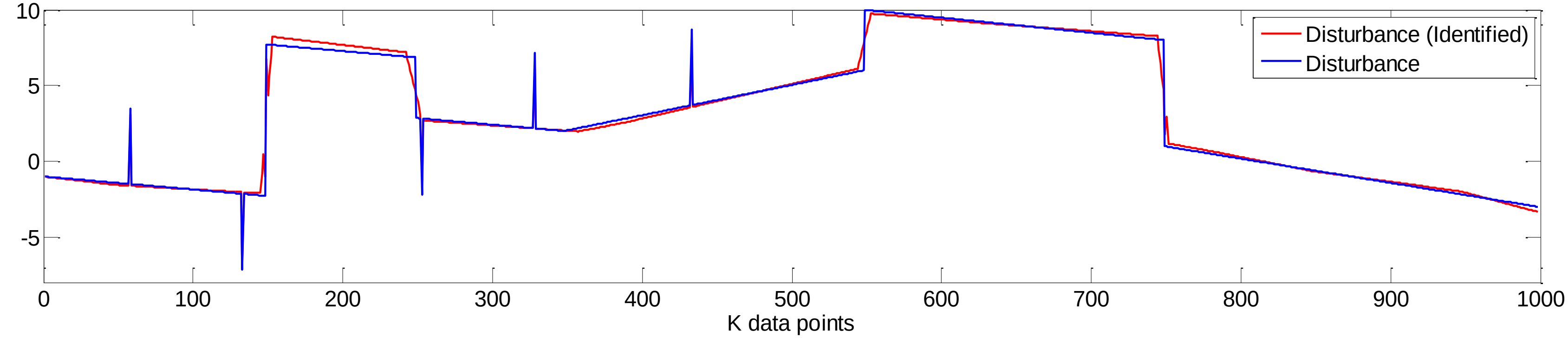
$\hat{\theta}$	1.5428	2.7305	1.3152	-0.8412	RMSE = 0.9102
$\hat{\theta}_{LS}$	1.5214	2.8084	1.3465	-0.7697	RMSE = 5.2937
without $d(k)$: RMSE $\hat{\theta}$ = 0.9125 $RMSE\ \hat{\theta}_{LS}$ = 0.9340					

Example1


Hankel Singular Values (State Contributions)
Stable modes
State Energy
State
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5

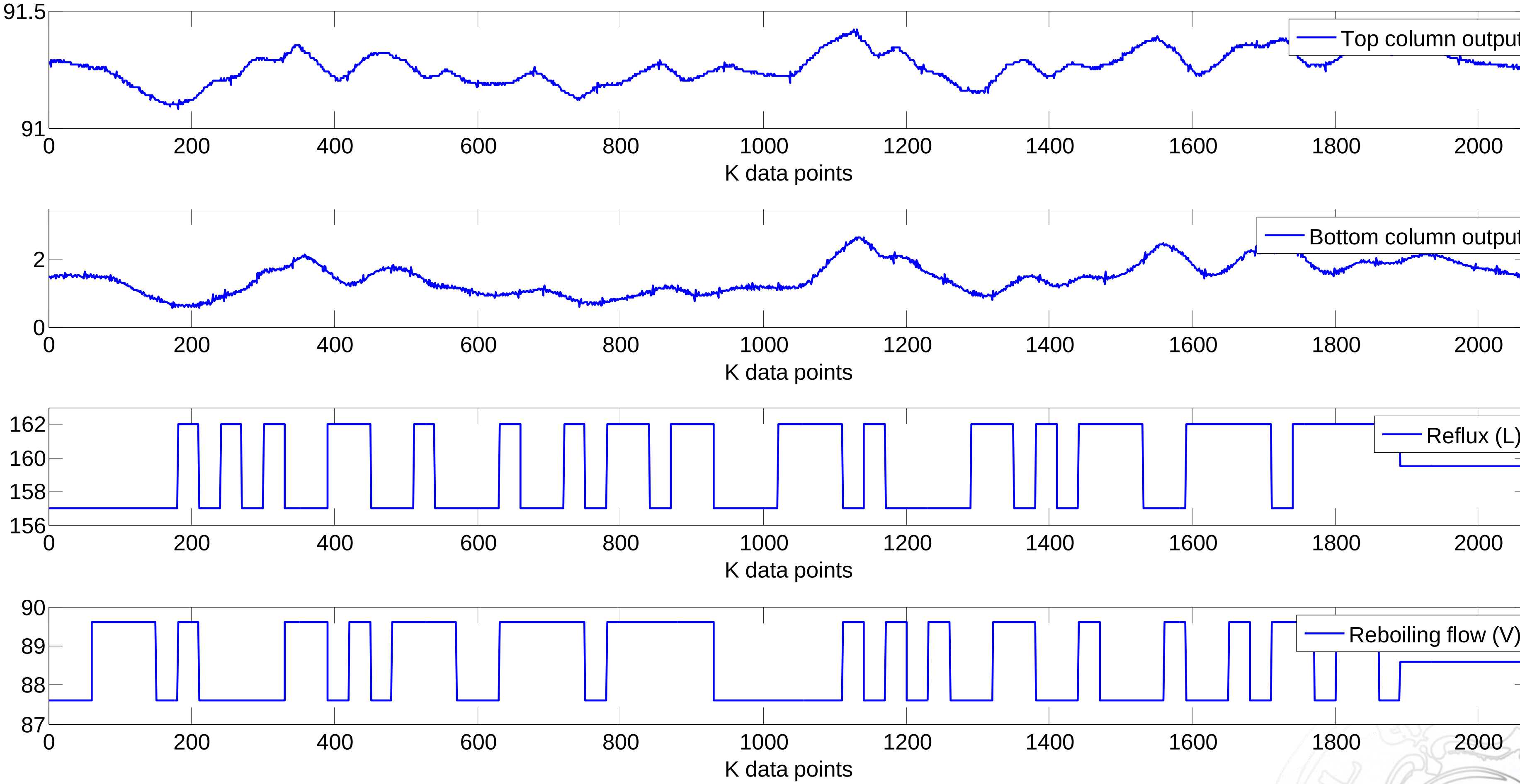
Example1



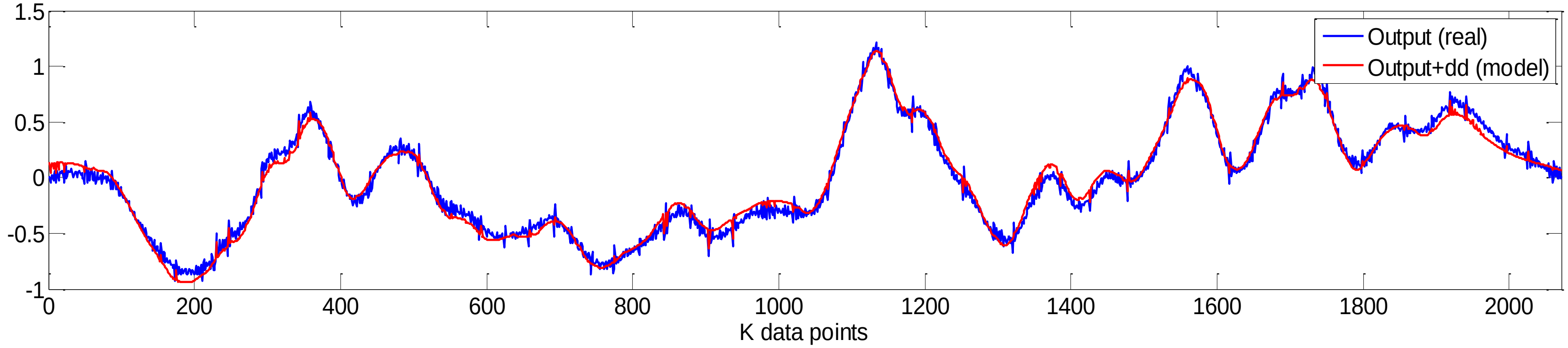


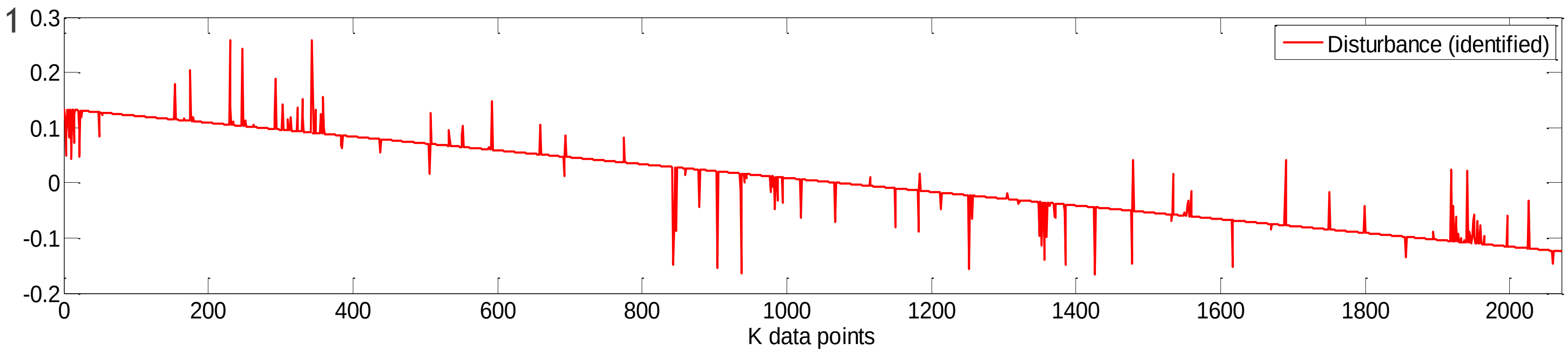
$\hat{\theta}$	1.5428 2.7305 1.3152 -0.8412	RMSE = 0.9102
$\hat{\theta}_R$	-0.6946 -0.4933	RMSE = 0.9118

Example2: Pilot-scale distillation column data



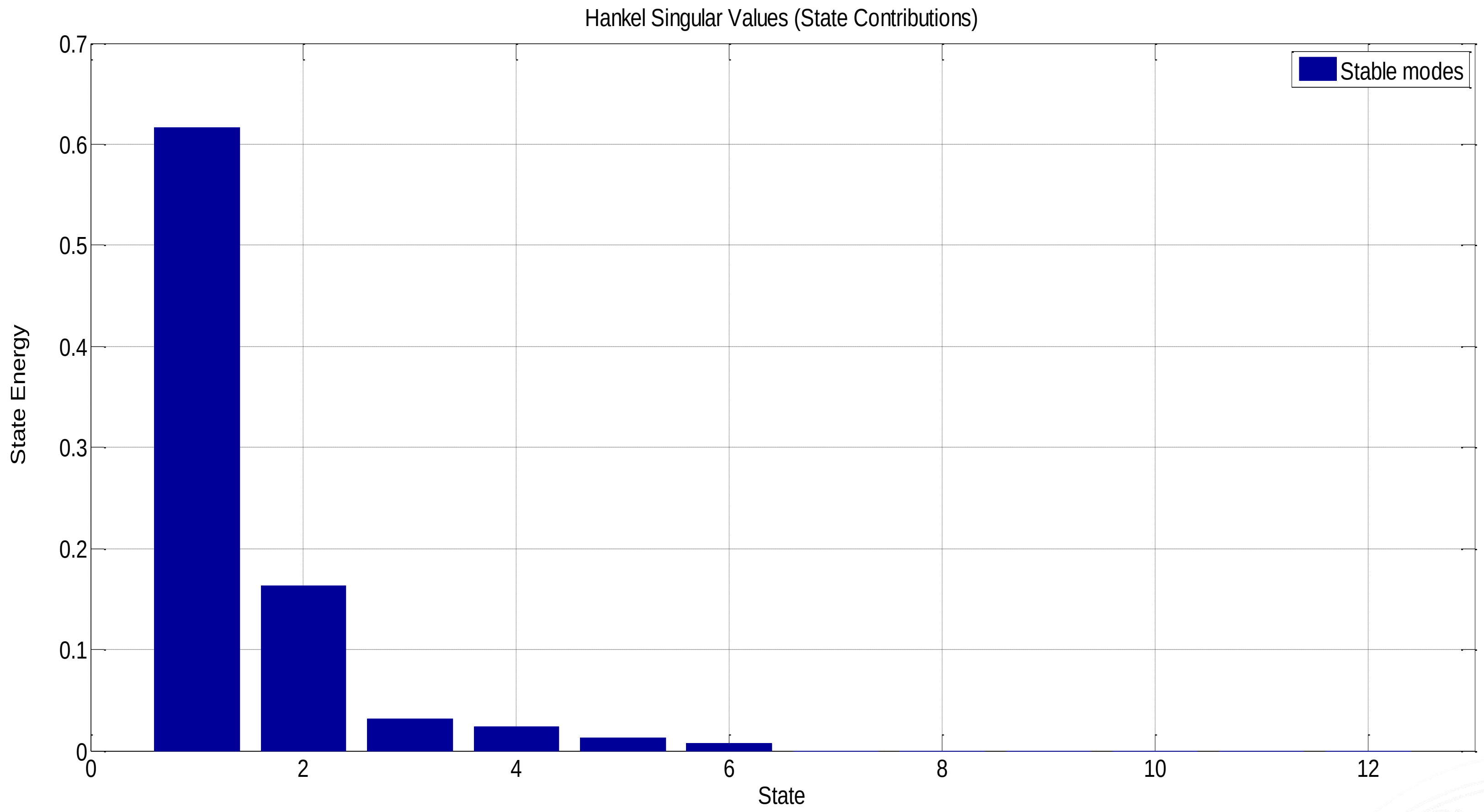
Example2: Identification results



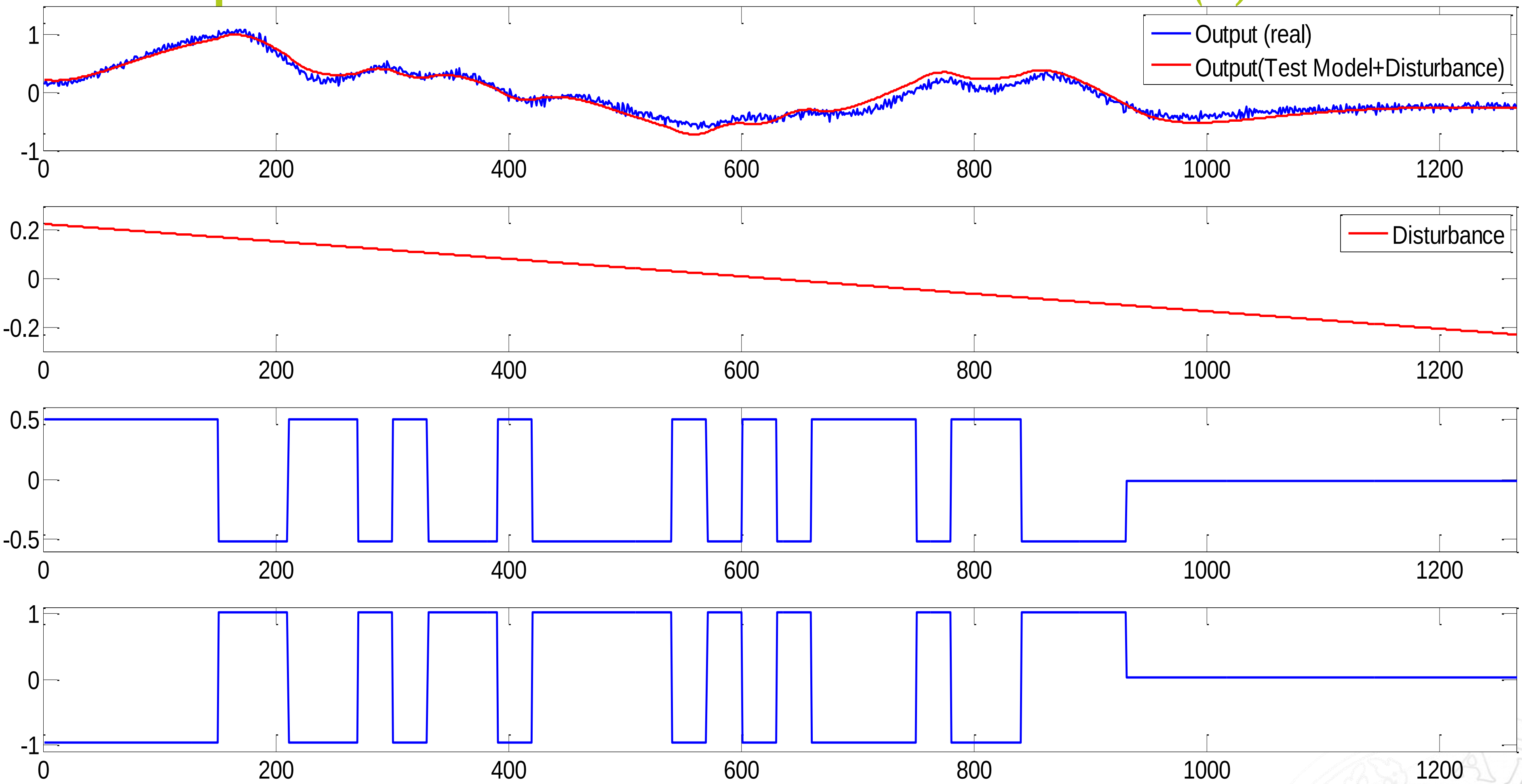


$\hat{\theta}$	$-0.0031 -0.0280 -0.0346 -0.0151 \quad \ldots (\text{total } 12)$	$\text{RMSE} = 0.0643$
$\hat{\theta}_{LS}$	$-0.0022 -0.0285 -0.0338 -0.0147 \quad \ldots (\text{total } 12)$	$\text{RMSE} = 0.0826$
$\hat{\theta}_{RED}$	$-0.0353 \quad -0.0456$	$\text{RMSE} = 0.0933$

Example2: Hankel singular value for reduction



Example2: Validation data with estimation of *d(k)*



$\hat{\theta}_{RED}$	$-0.0353 \quad -0.0456$	$\text{RMSE} = 0.0891$

Discussion and future work

Summary:

- Presented a method for identification of linear systems in the presence of structured disturbances (outliers, level shifts and trends) by using sparse optimization and orthogonal basis function as the system model
- Gives acceptable results for simulated example
- Gives acceptable results for distillation column example
- The orthogonal basis model improves the robustness and more insensitive to noise

Future work:

- Nonlinear system identification and trends and more general disturbances

Thank you for your attention!

Questions?