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A novel approach to include limited equipment connectivity in State-Task Network models

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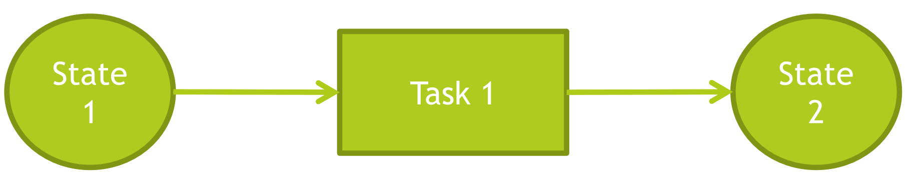
State Task Network models

- STN models
  - Are used for modeling batch process in both discrete and continuous time
- An STN graph consists of
  - Task nodes described as rectangles
  - State nodes described as circles
  - Directed arches
- States represent commodities
- Tasks transform one or more states into a new state
  - Tasks are preformed on units
- Arches describe batches of commodities moving through the graph

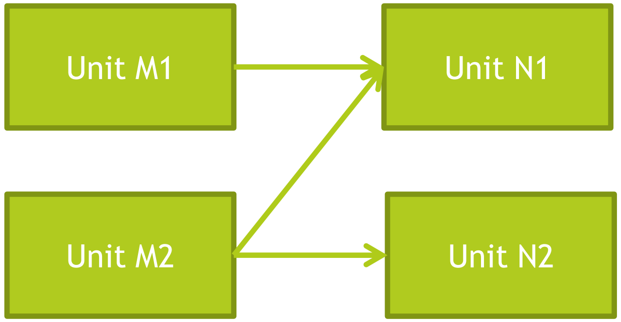
Example: A simple STN graph



Limited equipment connectivity

- STN graphs don’t explicitly include units in the graph
  - Units are included as an index in the mathematical model
- Limited equipment connectivity
  - When at least one unit in a production stage is not connected to all units in the next stage
  - Can only occure in multi stage systems
  - Common in many industries

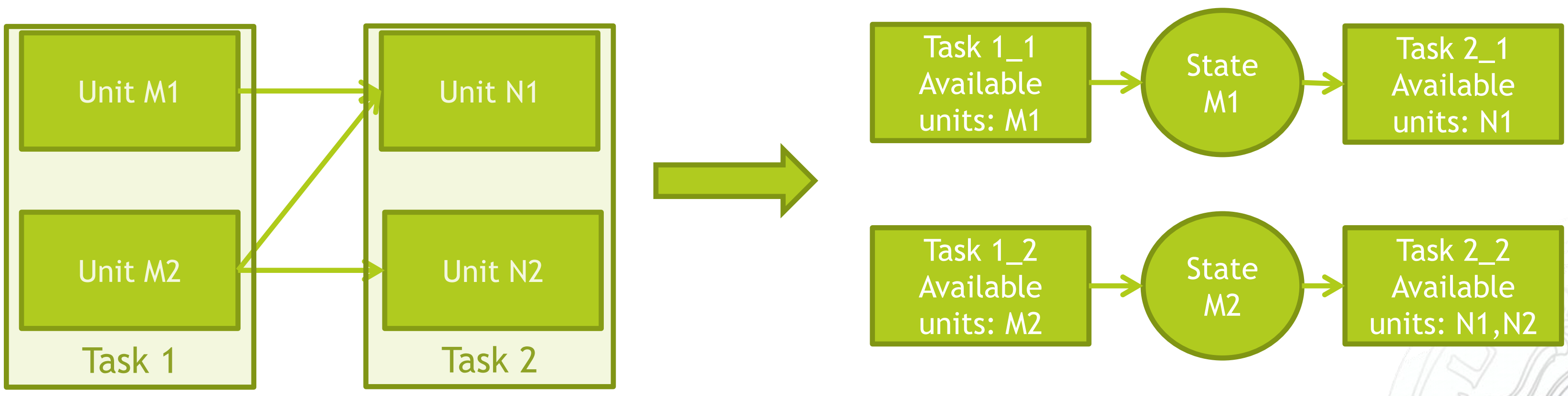
Example: Limited equipment connectivity



Task Splitting

- Kondili et al. proposed task splitting for including limited equipment connectivity in STN models
- Task Splitting does the following:
  - Duplicates tasks for units in the later stage to include limited connectivity
  - Task Splitting does not require any modifications to the mathematical model, only additional tasks are required
- The metod has two drawbacks:
  - Increases the number of binary variables
  - Prohibits merging of batches

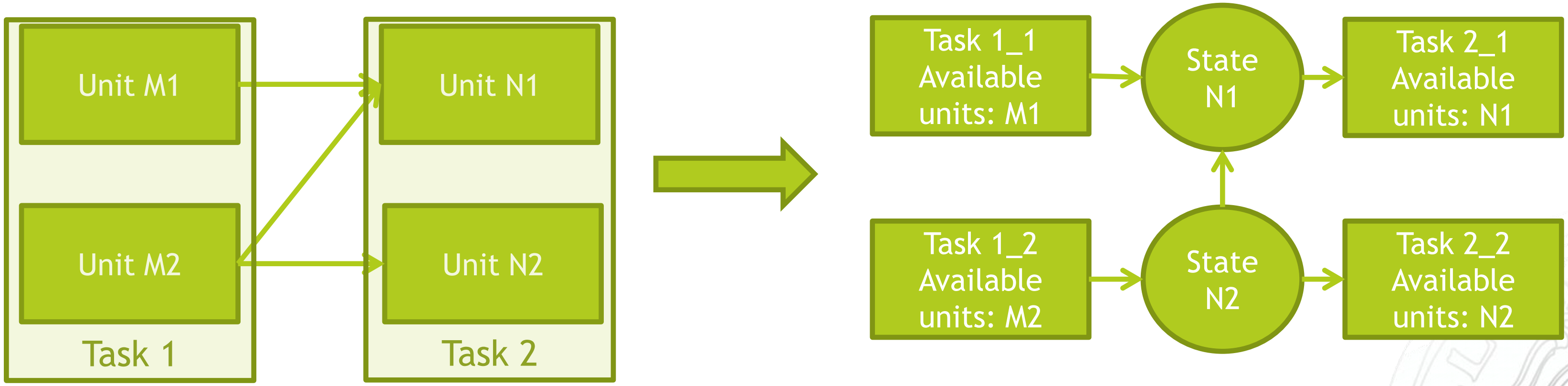
Example: Task Splitting



State splitting

- Includes limited connectivity by splitting states and divides batches into sub batches instead of splitting tasks
- Requires:
  - A new set of continuous variables
  - Reformulation of the mass balance constraint
  - Introduction of a new set of balance constraints

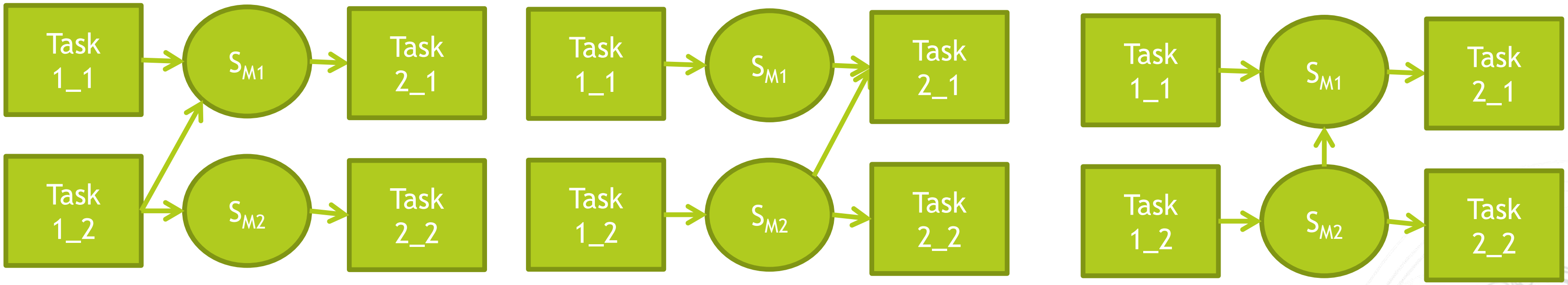
Example: State Splitting



State Splitting - three ways to do it

- For every unit configuration with limited connectivity State Splitting can be done in (at least) three ways:
  - Forward State Splitting (SS-F)
  - Backward State Splitting (SS-B)
  - Inter State Splitting (SS-I)
- The advantage of SS-F and SS-B are that one of them is always able to produce an STN model with a minimal number of additional variables
- The advantage of SS-I is that all limited equipment connectivity configurations are dealt with the same way
  - Reduces the modeling effort and simplifies the resulting mathematical model
- All three State Splitting methods can be mixed if need be

Example: State Splitting methods



SS-B 3 new var./time step

SS-F 3 new var./time step

SS-I 3 new var./time step

Mathematical formulation

- State Splitting for a discrete time STN model

New continuous variables

$S_{s,j',t}$ $\forall\, t, (s,j') \in {J_s}^s$ - amount of state *s* available for unit *j'*

$B_{i,j,j',t}$ $\forall\, t, j' \in {J_s}^s, j \in J_{j'}$ - size of batch *i* produced by unit *j* going to unit *j'*

Reformulated mass balance constraint

$$S_{s,j',t} = S_{s,j',t-1} + \sum_{i \in \bar{T}_s} \sum_{j \in J_{j'}} B_{i,j,j',t-p_{i,s}} - \sum_{i \in T_s} \sum_{j' \in K_i} B_{i,j',t} \quad \forall\, t, (s,j') \in {J_s}^s$$

New batch balance constraint

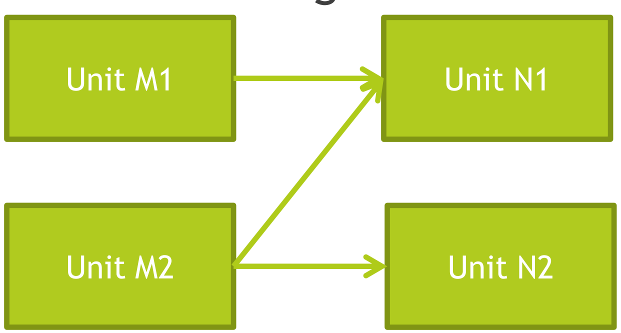
$$B_{i,j,t} = \sum_{j' \in J_j} B_{i,j,j',t} \quad \forall\, t, j, i \in {I_j}^s$$

Benefits of State Splitting

- State Splitting overcomes both drawback of Task Splitting as:
  - Only continuous variables are added to the model
    - The addition of only continuous variables reduces the increase in computation time compared to Task Splitting
    - The number of additional variables is roughly the same as for Task Splitting
  - Merging (and splitting) of batches is possible
    - The possibility to merge batches increases the flexibility of the mathematical model
    - In some cases this improves the solution quality
- The only draw back of State Splitting is a slight increase in model complexity

Comparison: Number of variables

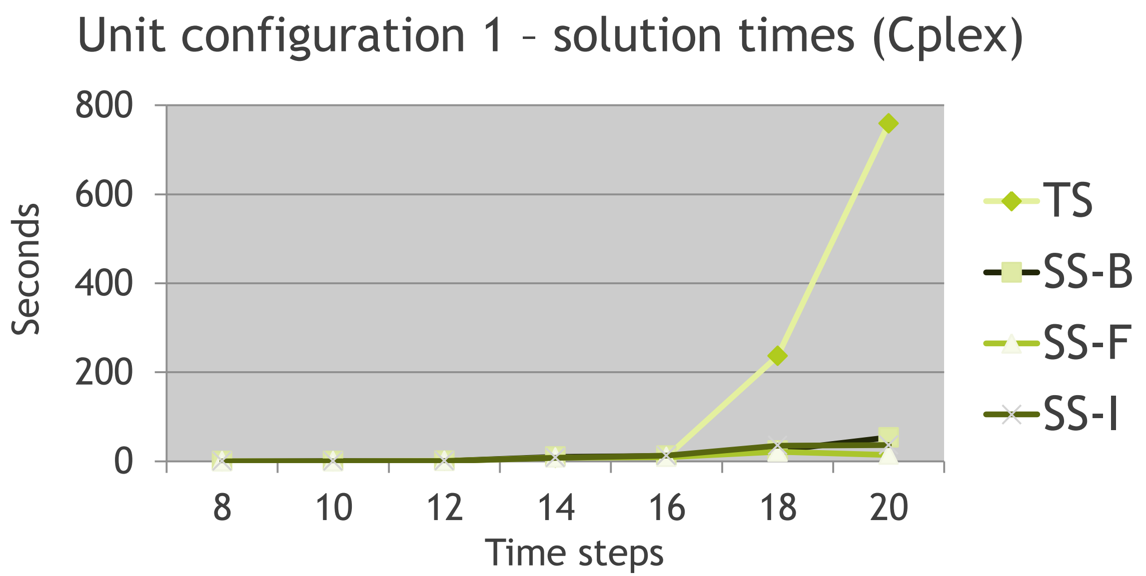
Unit configuration



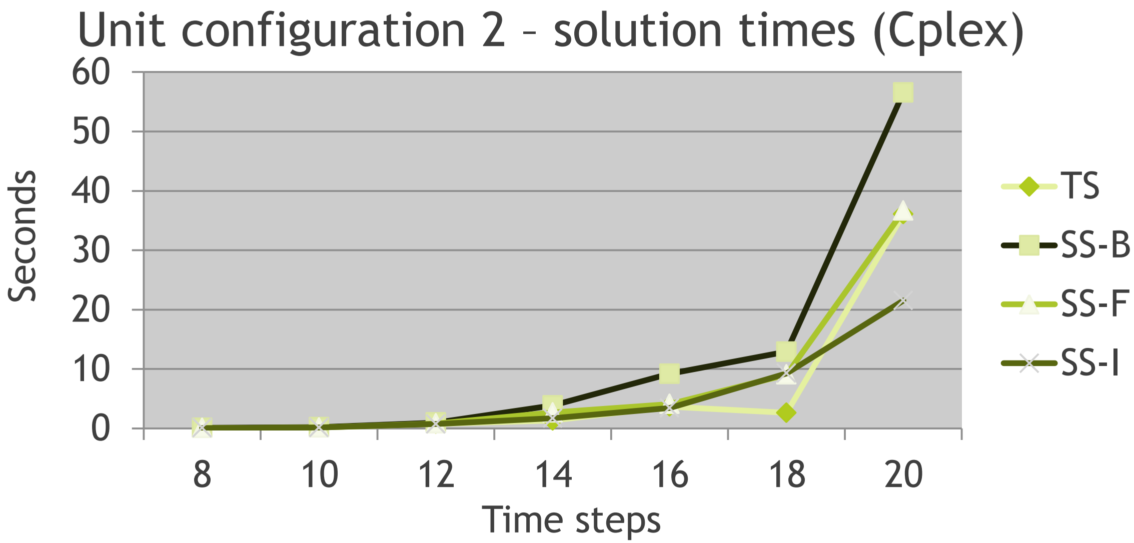
Method	# of binary *	# of cont. *
Task Splitting	5	19
State Splitting	4	16
*Number of variables / time step		

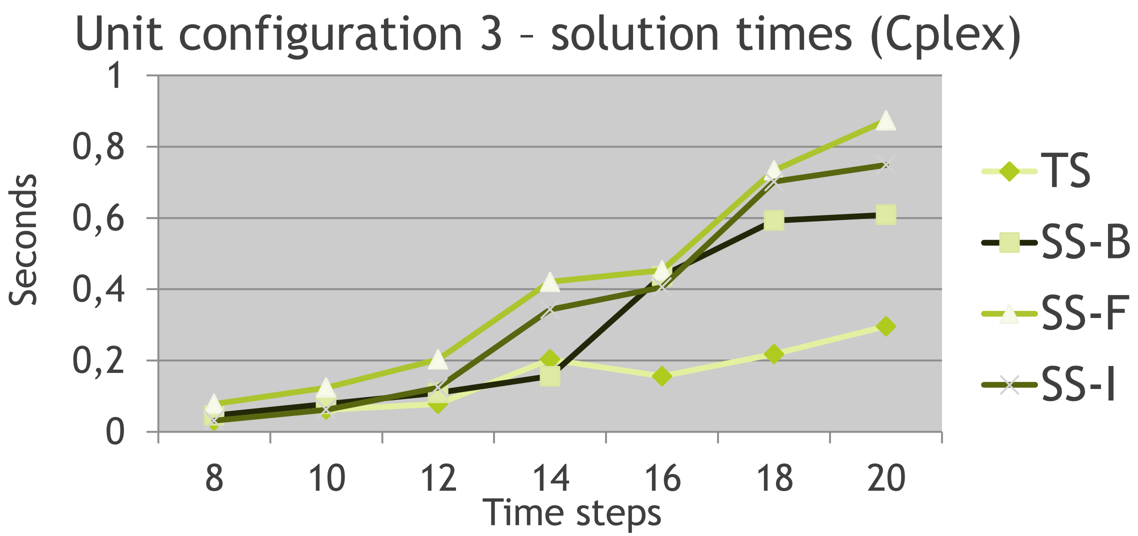
Computational results for a discrete time STN model using State Splitting

- A simple 2-commodity production planning problem
  - Objective function: maximize profit
  - 4 different unit configurations to test the different State Splitting methods compared to Task Splitting
  - Solved with 11 MILP solvers using GAMS 23.7



More results

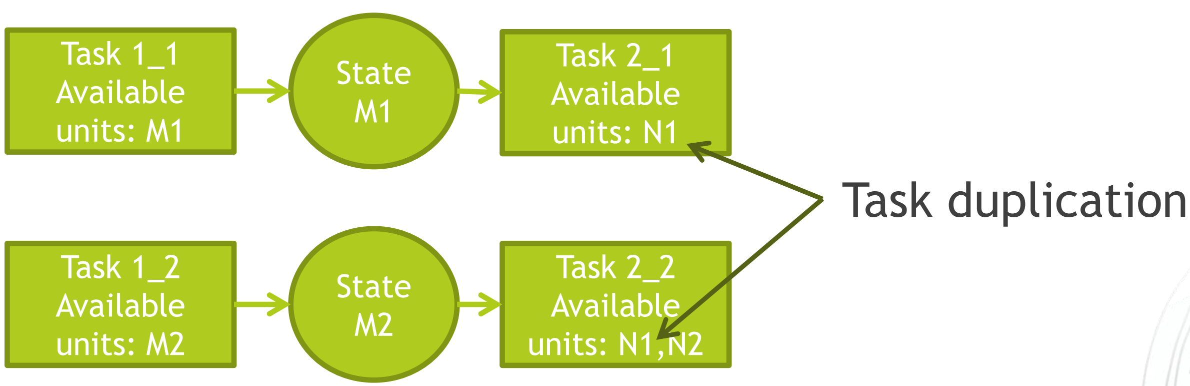




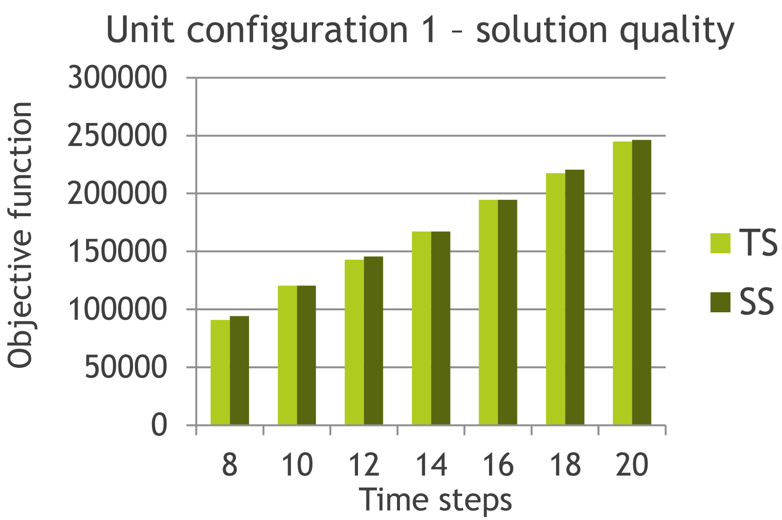
Improving solution quality

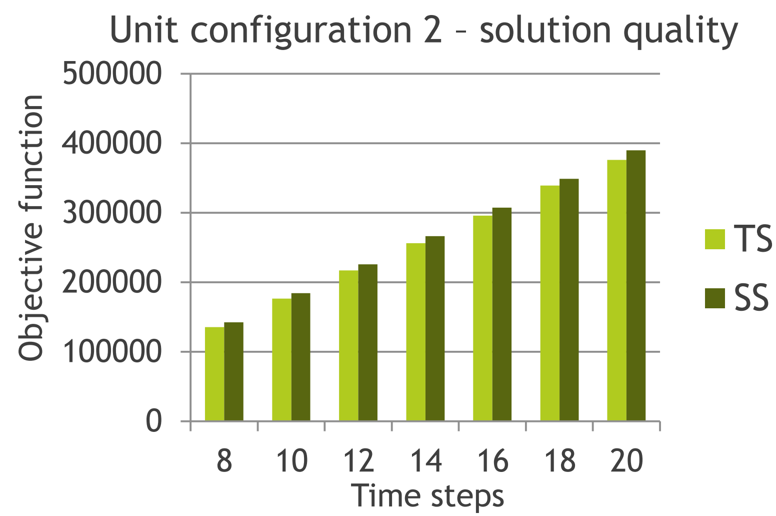
- State Splitting is able to produce better results than Task Splitting when:
  - The optimal solution includes at least one occurrence of batch merging
  - This is not possible in Task Splitting due to the following:
    - Tasks are duplicated
    - The allocation constraint only allows task to be executed on each unit
  - In State Splitting this can be done because no tasks have been duplicated

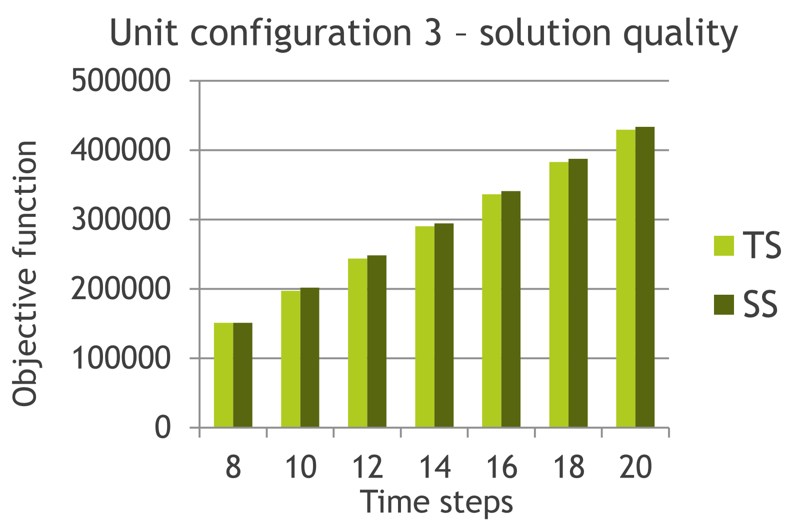
Example: Task duplication



Improving solution quality







Conclusions

- Solution times
  - Task Splitting vs. State Splitting
    - In all but one unit configuration at least one State Splitting method produced faster results than Task Splitting
    - In 50% of the cases all State Splitting methods were faster than Task Splitting
    - The results are consistent regardless of solver
  - SS-B vs. SS-F vs. SS-I
    - SS-I introduces more additional variables in all test cases compared to both SS-B and SS-F
      - This does not seem to impact solution efficiency
      - Good news as SS-I is easier to implement than SS-B or SS-F

Conclusions

- Solution quality
  - Task Splitting vs. State Splitting
    - In 81% of all test problems State Splitting produced better results than Task Splitting
    - The average improvement was 2,01%
  - SS-B vs. SS-F vs. SS-I
    - Solution quality is independent of State Splitting method

- So far the performance of State Splitting is very promising both in regards of solution speed and quality
  - More tests are needed to before any general conclusion can be made concerning improvements solving efficiency
  - The improvements in solution quality are certain

The future

- Continue testing the State Splitting approach on:
  - Large scale discrete time STN problems
  - Small and large scale continuous time STN problems
- Investigate if State Splitting can be used in some of the following areas:
  - Zero Wait production planning problems
  - Detailed storage modeling
  - Blocking

Thank you!

Questions?