Discrete and Continuous Optimization Models for the Design and Operation of Sustainable and Robust Process Systems

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Motivation

1. Increasing interest in **energy systems and supply chains**

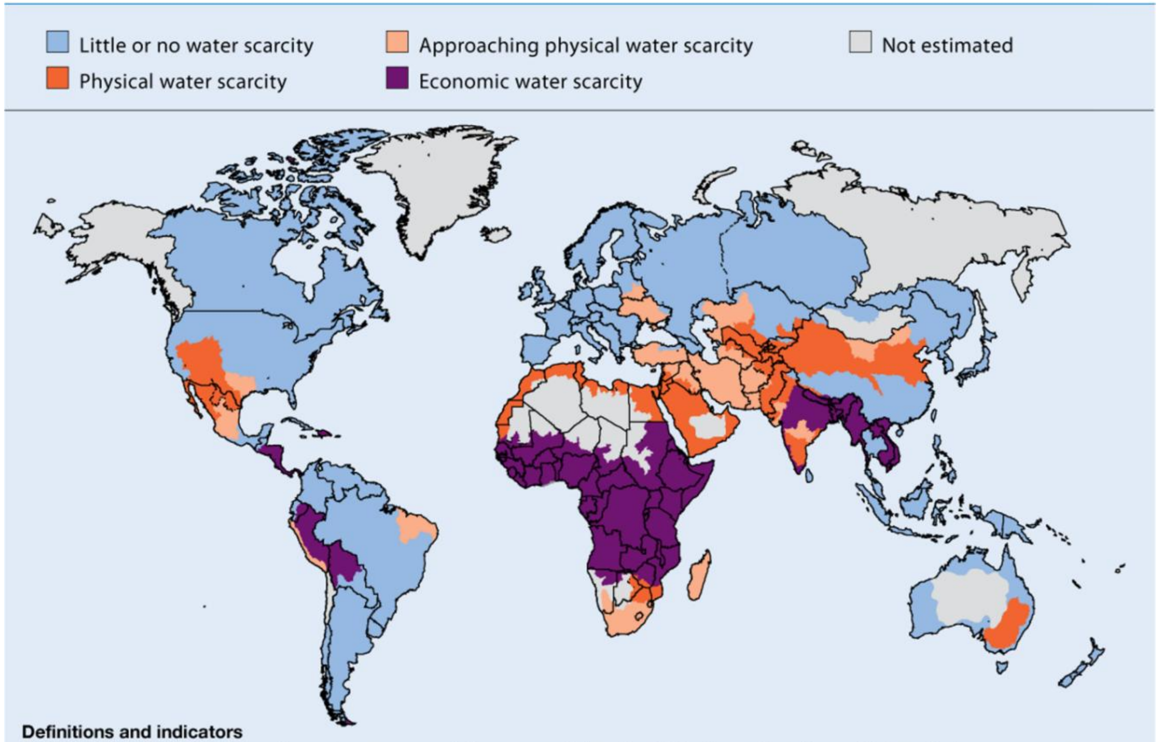
2. Need to address design of **sustainable chemical processes**
   - Minimize **energy use**
   - Minimize **water consumption**

3. Need to introduce robustness to account for **uncertainties**

Goal: Systematic Optimization Approaches for **Sustainable and Robust** Optimization Process Design and PlanningOperations Problems

Challenges: *Nonconvexities in MINLP/GDP models*
Large-scale stochastic optimization problems

Water scarcity



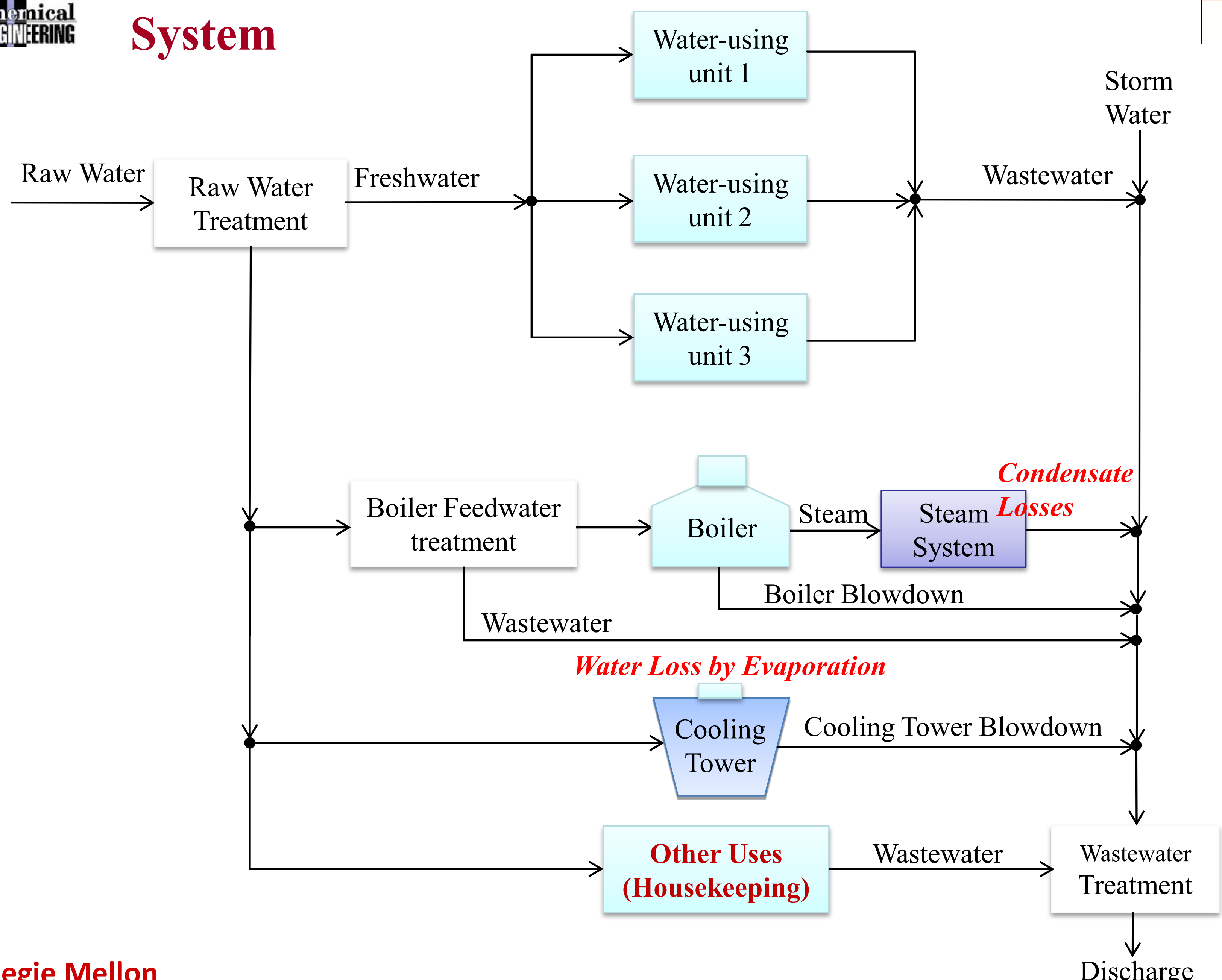
Definitions and indicators

- *Little or no water scarcity.* Abundant water resources relative to use, with less than 25% of water from rivers withdrawn for human purposes.
- *Physical water scarcity (water resources development is approaching or has exceeded sustainable limits).* More than 75% of river flows are withdrawn for agriculture, industry, and domestic purposes (accounting for recycling of return flows). This definition—relating water availability to water demand—implies that dry areas are not necessarily water scarce.
- *Approaching physical water scarcity.* More than 60% of river flows are withdrawn. These basins will experience physical water scarcity in the near future.
- *Economic water scarcity (human, institutional, and financial capital limit access to water even though water in nature is available locally to meet human demands).* Water resources are abundant relative to water use, with less than 25% of water from rivers withdrawn for human purposes, but malnutrition exists.

Source: International Water Management Institute analysis done for the Comprehensive Assessment of Water Management in Agriculture using the Watersim model; chapter 2.

Two-thirds of the world population will face water stress by year 2025

Conventional System



Synthesis of Integrated Process Water Networks

- **Given is:**
  - a set of single/multiple water sources with/without contaminants,
  - a set of water-using, water pre-treatment, and wastewater treatment operations, sinks and sources of water

- **Synthesize** an integrated process water network
  - interconnection of process and treatment units (reuse, recycle)
  - the flow rates and contaminants concentration of each stream
  - minimum total annual cost of water network

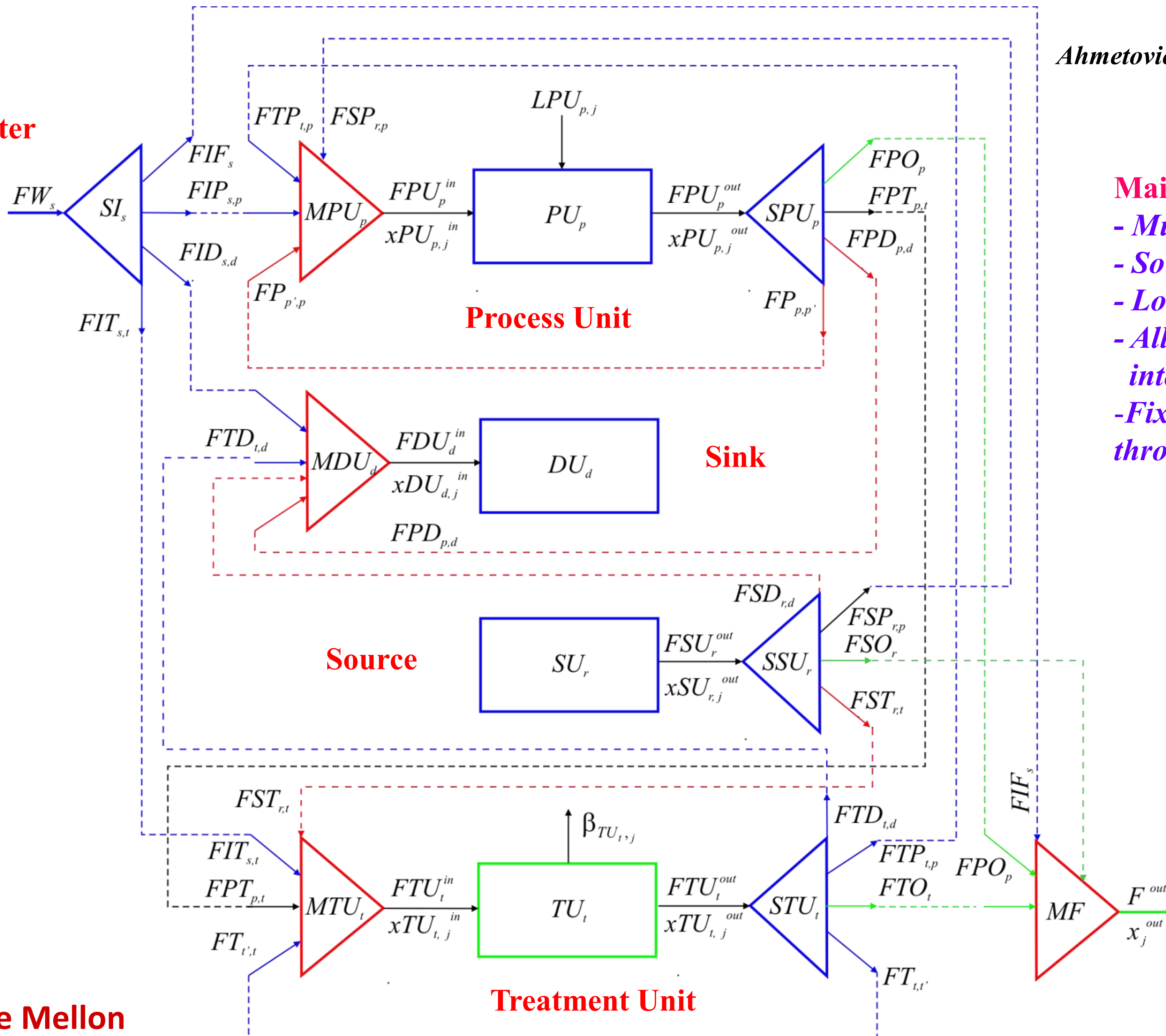
Synthesis Integrated Process Water Networks

- Pinch analysis and mathematical programming models
- *Reviews in Bagajewicz (2000), Ježowski (2008), Bagajewicz and Faria (2009), and Foo (2009).*

Approach: Global NLP or MINLP superstructure optimization model

Superstructure for water networks for water reuse, recycle, treatment, and with sinks/sources water

Ahmetovic, Grossmann (2010)



Main features:

- *Multiple feeds*
- *Source/Sink units*
- *Local recycles*
- *All possible interconnections*
- *Fixed and variable flows through process units*

Optimization Model

Nonconvex NLP or MINLP

Objective function: ***min Cost***

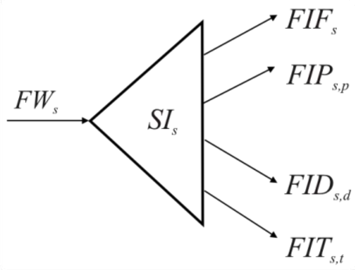
Subject to:

Splitter mass balances
Mixer mass balances (bilinear)
Process units mass balances
Treatment units mass balances
Design constraints

0-1 variables for piping sections

Model can be solved to global optimality

Splitter



$$FW_s = FIF_s + \sum_{p \in PU} FIP_{s,p} + \sum_{d \in DU} FID_{s,d} + \sum_{t \in TU} FIT_{s,t} \qquad \forall s \in SW$$

linear

$$FIF_s^L \cdot y_{FIF_s} \le FIF_s \le FIF_s^U \cdot y_{FIF_s} \qquad \forall s \in SW$$

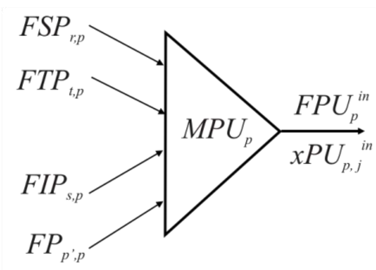
$$FIP_{s,p}^L \cdot y_{FIP_{s,p}} \le FIP_{s,p} \le FIP_{s,p}^U \cdot y_{FIP_{s,p}} \qquad \forall s \in SW, \forall p \in PU$$

$$FID_{s,d}^L \cdot y_{FID_{s,d}} \le FID_{s,d} \le FID_{s,d}^U \cdot y_{FID_{s,d}} \qquad \forall s \in SW, \forall d \in DU$$

$$FIT_{s,t}^L \cdot y_{FIT_{s,t}} \le FIT_{s,t} \le FIT_{s,t}^U \cdot y_{FIT_{s,t}} \qquad \forall s \in SW, \forall t \in TU$$

0-1 optional

Mixer



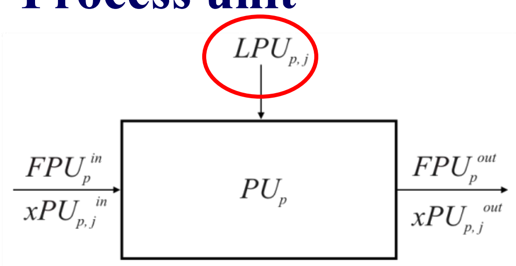
$$FPU_p^{in} = \sum_{r \in SU} FSP_{r,p} + \sum_{t \in TU} FTP_{t,p} + \sum_{s \in SW} FIP_{s,p} + \sum_{\substack{p' \in PU \\ p \ne p', R_p = 0}} FP_{p',p} + \sum_{\substack{p' \in PU \\ R_p = 1}} FP_{p',p}, \qquad \forall p \in PU$$

$$FPU_p^{in} \cdot xPU_{p,j}^{in} = \sum_{r \in SU} FSP_{r,p} \cdot xSU_{r,j}^{out} + \sum_{t \in TU} FTP_{t,p} \cdot xSTU_{t,j}^{out} + \sum_{s \in SW} FIP \cdot xW_{s,j}^{in}$$

bilinear

$$+ \sum_{\substack{p' \in PU \\ p \ne p', R_p = 0}} FP_{p',p} \cdot xSPU_{p',j}^{out} + \sum_{\substack{p' \in PU \\ R_p = 1}} FP_{p',p} \cdot xSPU_{p',j}^{out}, \qquad \forall p \in PU, \forall j$$

Process unit



$$FPU_p^{in} = FPU_p^{out} \qquad \forall p \in PU$$

linear if flowrate is fixed

$$FPU_p^{in} \cdot xPU_{p,j}^{in} + LPU_{p,j} \cdot 10^3 = FPU_p^{out} \cdot xPU_{p,j}^{out} \qquad \forall p \in PU, \forall j$$

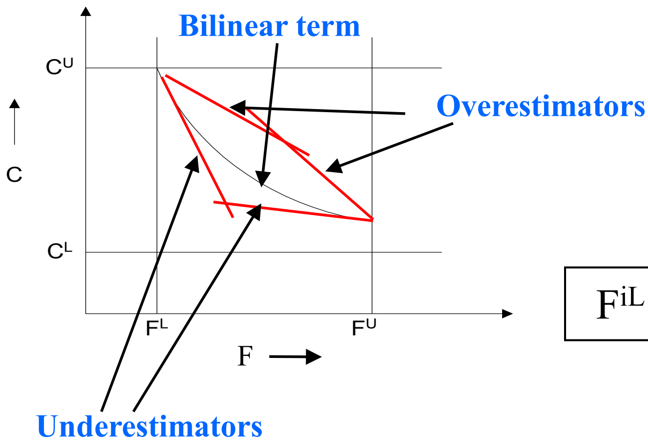
bilinear if the flow treated as cont. variable

Cost function linear in feedwater, concave in treatment unit, linear in operating cost, pipe section fixed charge (0-1)

$$\min Z = H \cdot \sum_{s \in SW} FW_s \cdot CFW_s + AR \cdot \sum_{t \in TU} IC_t \cdot \left(FTU_t^{out}\right)^{\alpha} + H \cdot \sum_{t \in TU} OC_t \cdot FTU_t^{out}$$

Convexification of Non-convex functions

Convex Envelopes for Bilinear Terms ***F*C***



$$
\begin{aligned}
f_j^i &\ge F^{iL} C_j^i + C_j^{iL} F^i - F^{iL} C_j^{iL} \\
f_j^i &\ge F^{iU} C_j^i + C_j^{iU} F^i - F^{iU} C_j^{iU} \\
f_j^i &\le F^{iL} C_j^i + C_j^{iU} F^i - F^{iL} C_j^{iU} \\
f_j^i &\le F^{iU} C_j^i + C_j^{iL} F^i - F^{iU} C_j^{iL}
\end{aligned}
$$

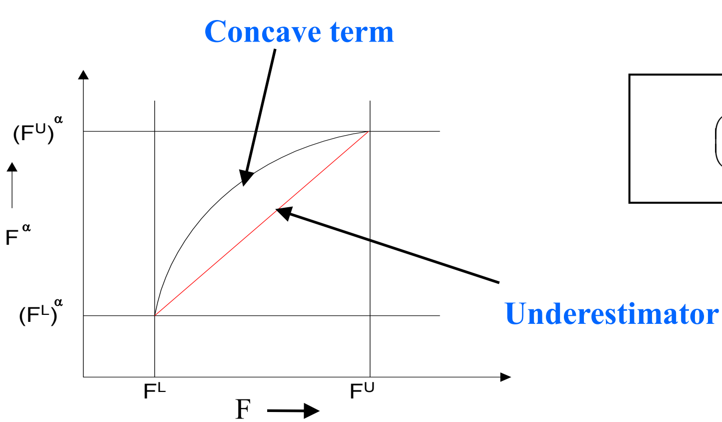
McCormick (1976)

Under- and over-estimators

(Linear Inequalities)

$F^{iL} \le F^i \le F^{iU}$ $C_j^{iL} \le C_j^i \le C_j^{iU}$

Underestimation of Concave functions



$$
\left(\overline{F}^i\right) \ge \left(F^{iL}\right)^{\alpha} + \left(\frac{\left(F^{iU}\right)^{\alpha} - \left(F^{iL}\right)^{\alpha}}{F^{iU} - F^{iL}}\right) \times \left(F^i - F^{iL}\right)
$$

(Secant line)

$F^{iL} \le F^i \le F^{iU}$

•The cut proposed by Karuppiah and Grossmann (2006) is incorporated to significantly improve the strength of the lower bound for the global optimum: ***contaminant flow balances for the overall water network system***

$$\sum_{s\in SW} FW_s \cdot xW_{s,j}^{in} + \sum_{p\in PU} LPU_{p,j} \cdot 10^3 + \sum_{r\in SU} FSU_r^{out} \cdot xSU_{r,j}^{out} = \sum_{t\in TU} (1-\beta_{TU_{t,j}}) \cdot FTU_t^{in} \cdot xTU_{t,j}^{in} + F^{out} \cdot x_j^{out} + \sum_{d\in DU} FDU_d^{in} \cdot xDU_{d,j}^{in} \qquad \forall j$$

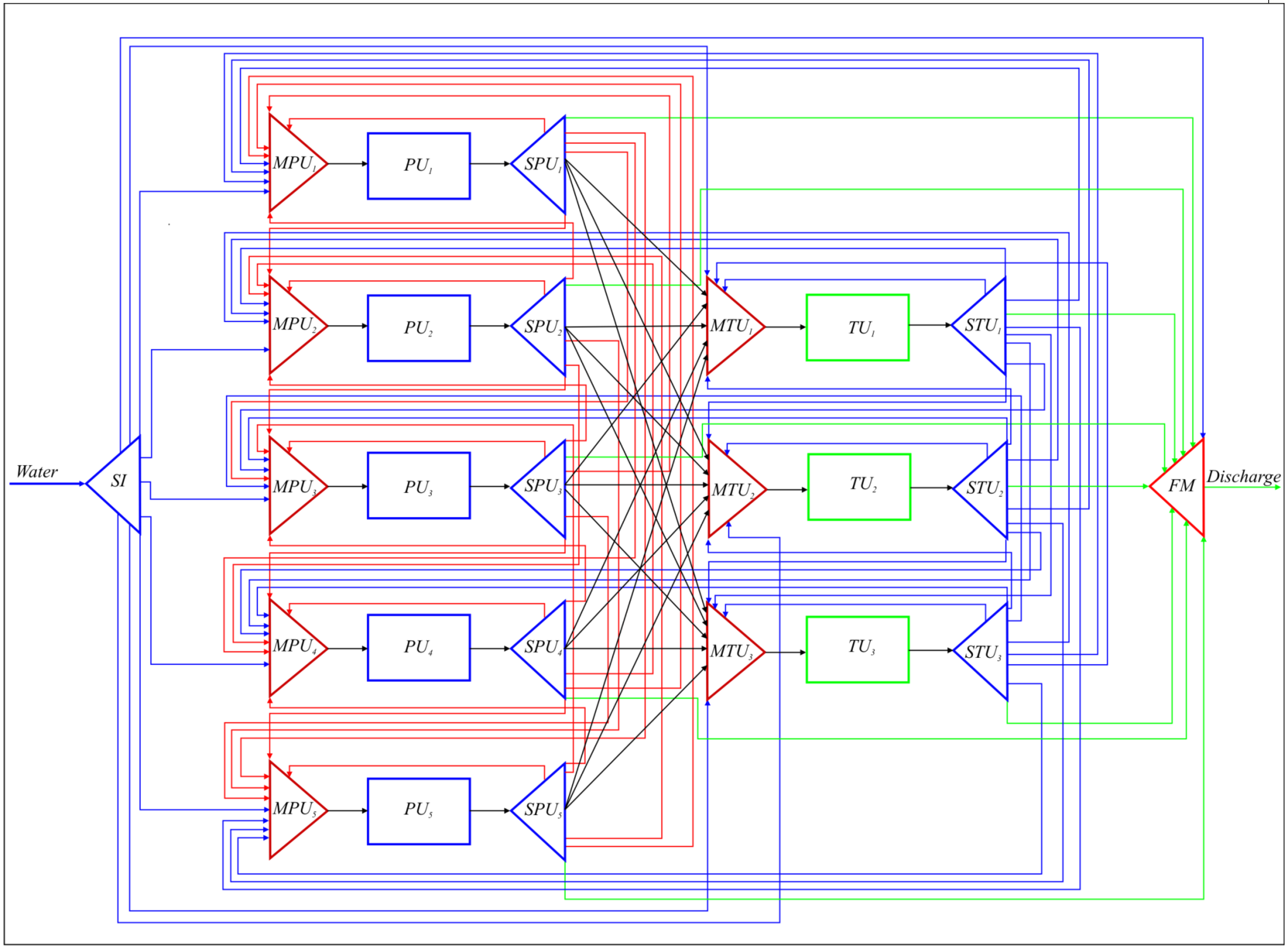
bilinear terms for the treatment units and final mixing points

Cut is redundant for original problem Non-redundant for relaxation problem

•Tight bounds on the variables are expressed as general equations obtained by physical inspection of the superstructure and using logic specifications

Superstructure of the integrated water network

1 feed, 5 process units, 3 treatment units, 3 contaminants



MINLP: 72 0-1 vars, 233 cont var, 251 constr
BARON optcr=0.01 197.5 CPUsec

Optimal design of the simplified water network with 13 removable connections

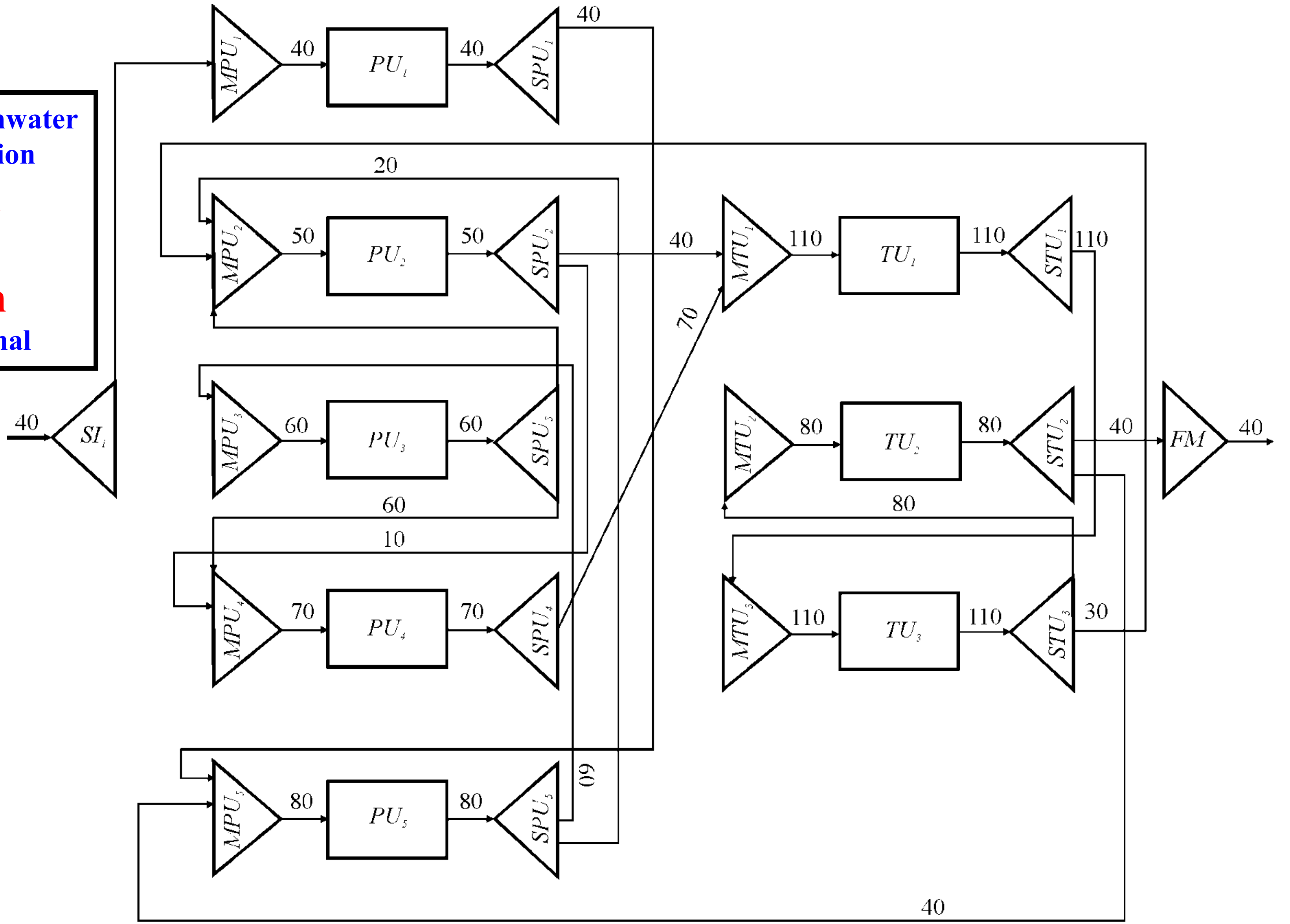
Optimal Freshwater Consumption

40 t/h

vs

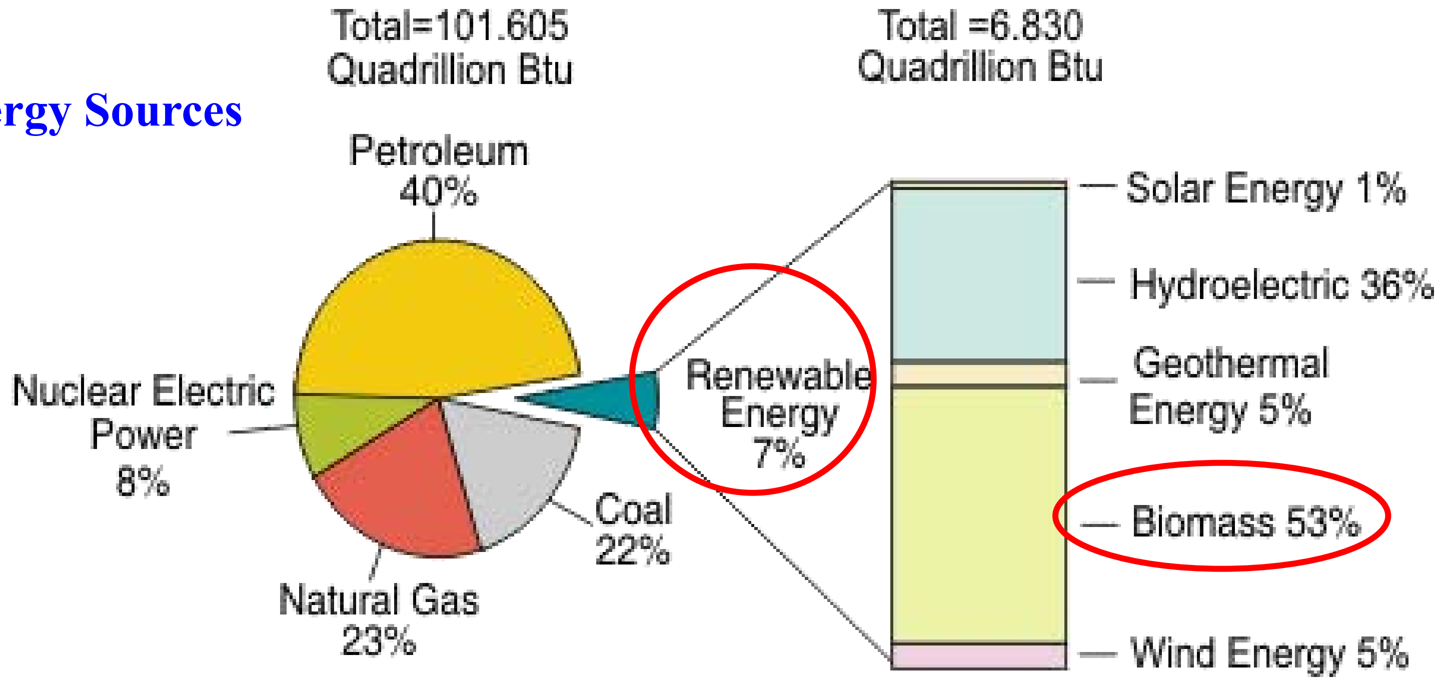
300 t/h

conventional



Biomass emerging as important renewable

US Energy Sources



Note: Sum of components may not equal 100 percent due to independent rounding.
Source: EIA, *Renewable Energy Consumption and Electricity Preliminary 2007 Statistics*, Table 1: U.S. Energy Consumption by Energy Source, 2003-2007 (May 2008).

Process Design Challenges in Bioethanol

Energy consumption corn-based process level:

Author (year)	Energy consumption (Btu/gal)
Pimentel (2001)	75,118
Keeney and DeLuca (1992)	48,470
Wang et al. (1999)	40,850
Shapouri et al. (2002)	51,779
Wang et al (2007)	<u>38,323</u>

Water consumption corn based - process level:

Author (year)	Water consumption (gal/gal ethanol)
Gallager (2005) First plants	11
Philips (1998)	5.8
MATP (2008) Old plants in 2006	4.6
MATP (2008) New plants	<u>3.4</u>

Proposed Design Strategy for Energy and Water Optimization

Energy optimization

Issue: fermentation reactions at modest temperatures

=> No source of heat at high temperature as in petrochemicals

Multieffect distillation followed by heat integration process streams

Water optimization

Issue: cost contribution is currently still very small (freshwater contribution < 0. 1%)

=> Total cost optimization is unlikely to promote water conservation

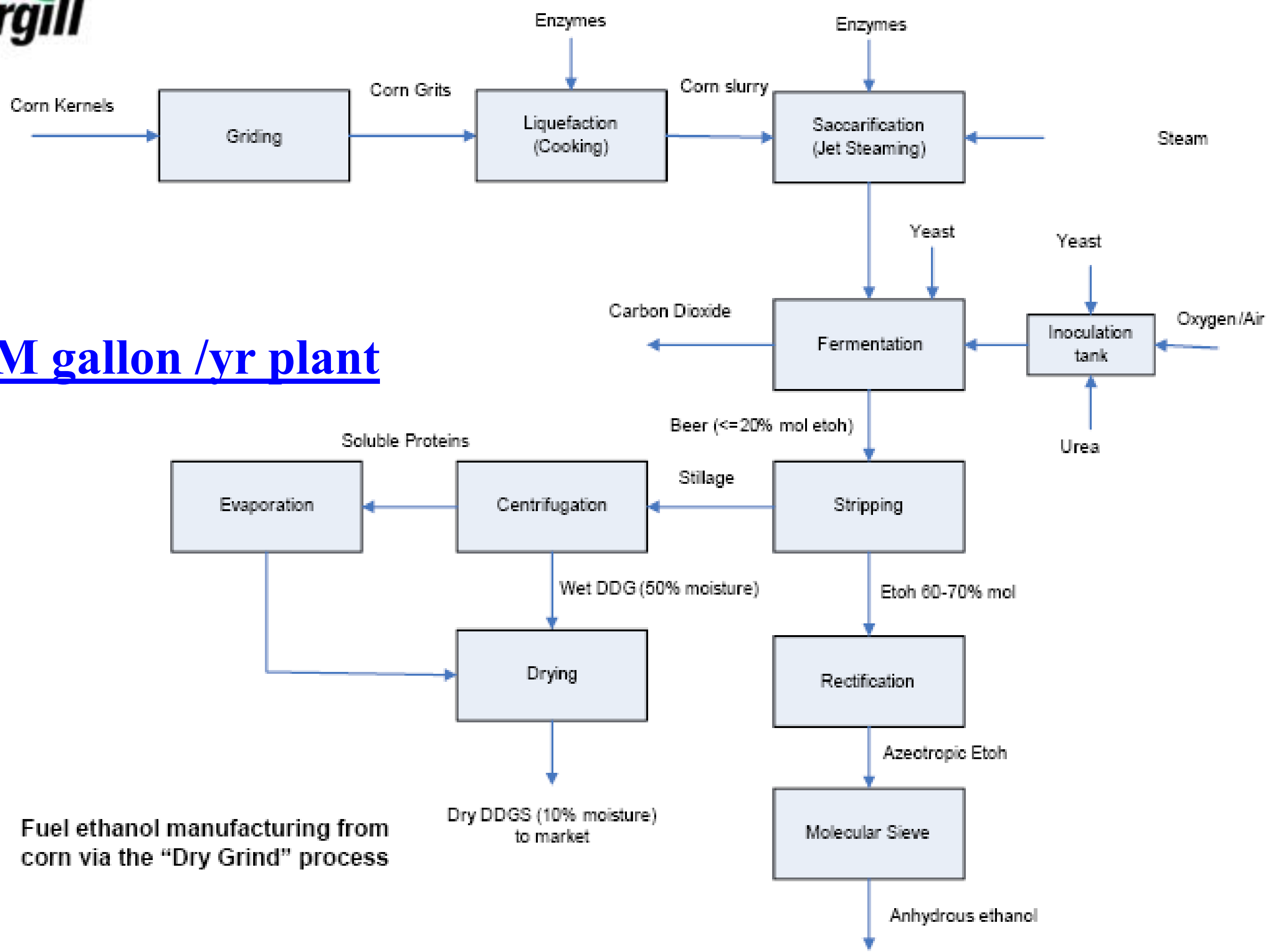
Optimal process water networks for minimum energy consumption

Energy Optimization of Corn-based Bioethanol

Peschel, Martin, Karuppiah, Grossmann, Zullo, Martinson (2007)

Cargill

60 M gallon /yr plant



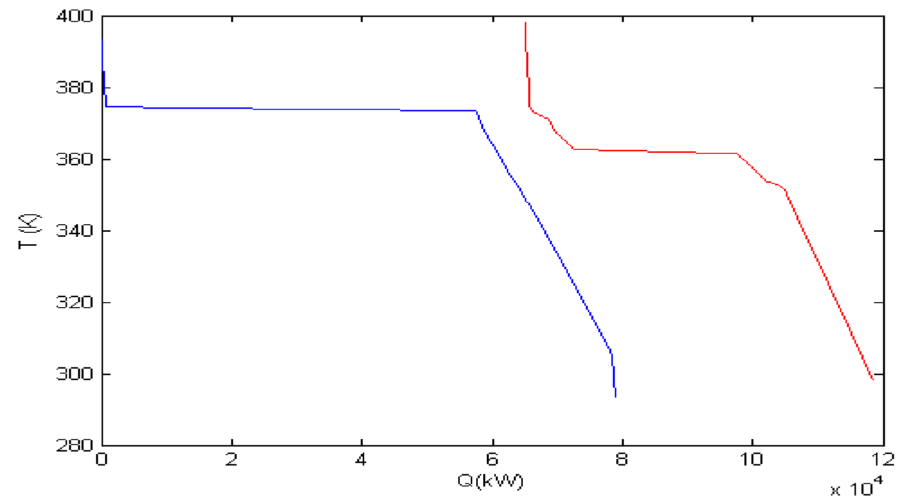
Fuel ethanol manufacturing from corn via the "Dry Grind" process

Equipment cost = M$ 18.4 **Steam cost = M$ 21/yr**

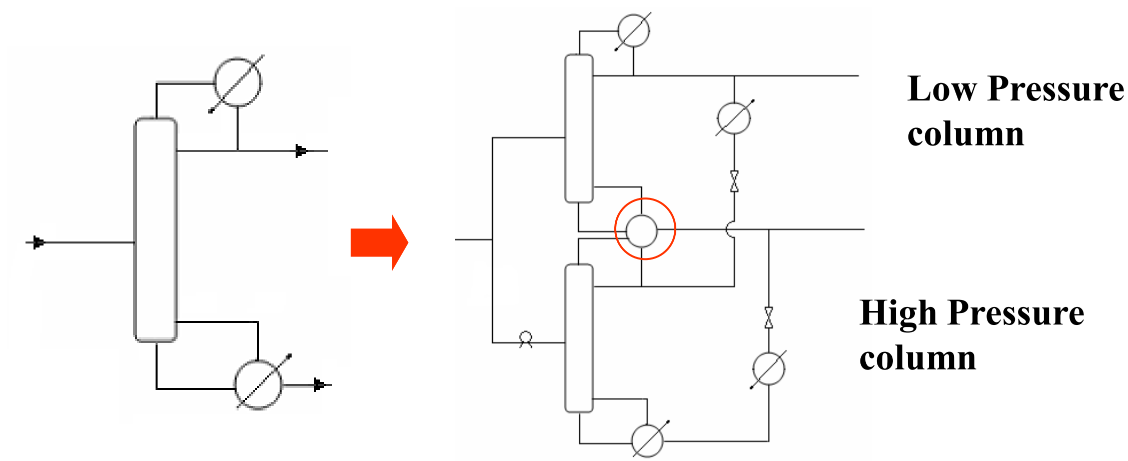
Prod. cost = 1.50 $/gal

Alternatives for Energy Reduction

Heat Integration process streams:



Multieffect columns:

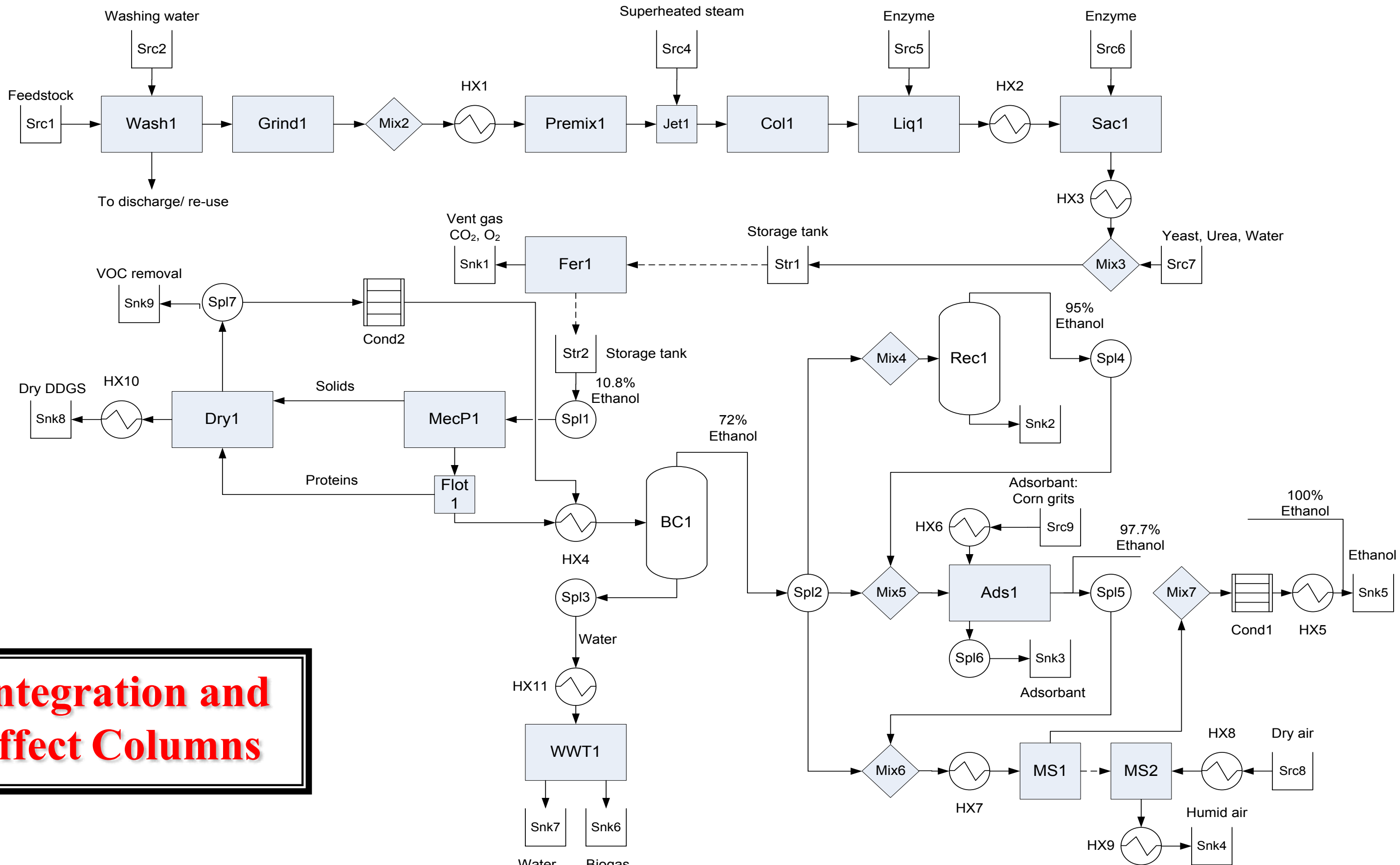


GDP model comprises mass, energy balances, design equations *(short cut)*

2,922 variables (2 Boolean) 2,231 constraints

Energy Optimal Design

60 M gallon /yr plant



Heat Integration and Multieffect Columns

Ethanol losses : 0.5%

Equipment cost = **M$ 20.7**

Steam cost = **M$ 7.1/yr (-66%)**

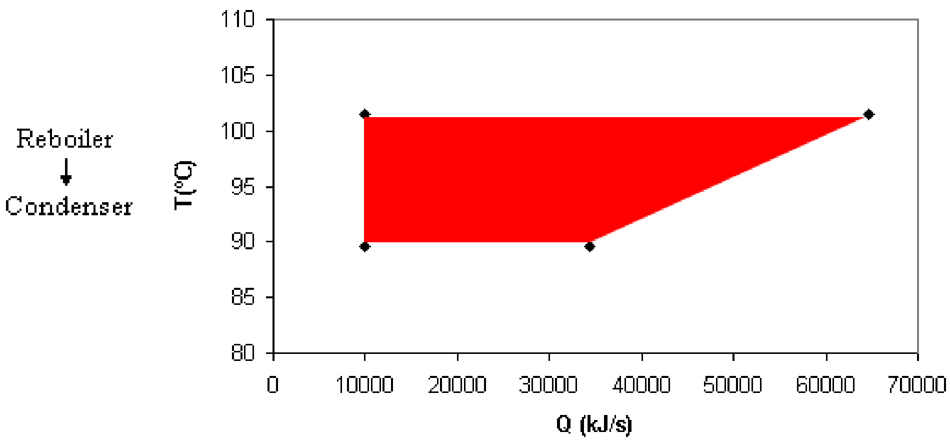
Prod. cost = 1.28 **$/gal**

Reduction from $1.50/gal (base case) to $1.28/gal !

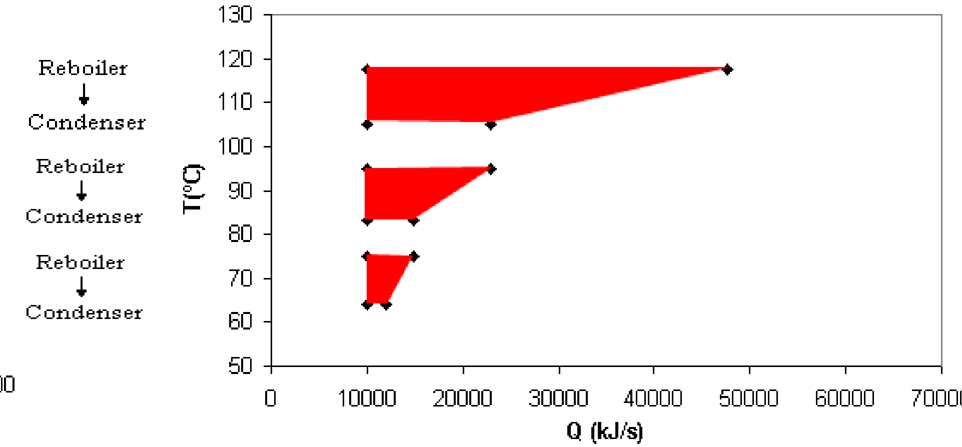
Energy Profiles in Multieffect Columns

Beer Column

Single column

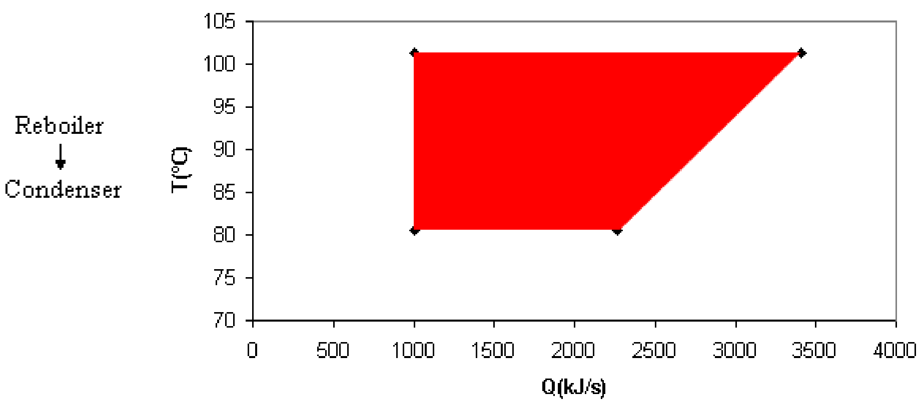


Triple effect column

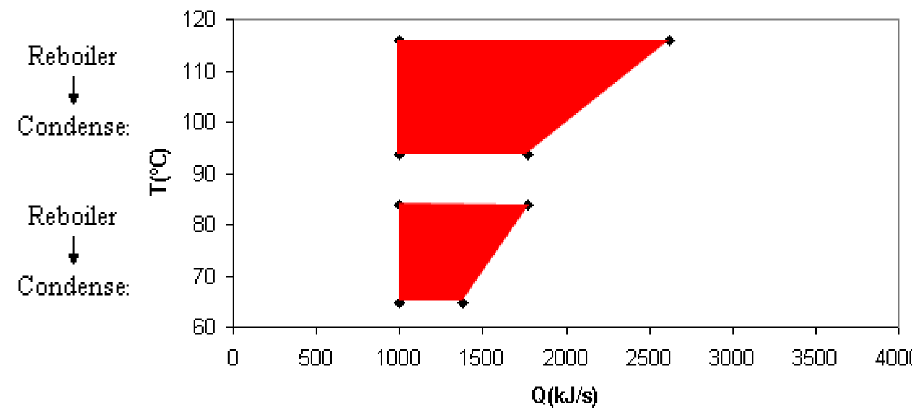


Rectification Column

Single column



Double effect column

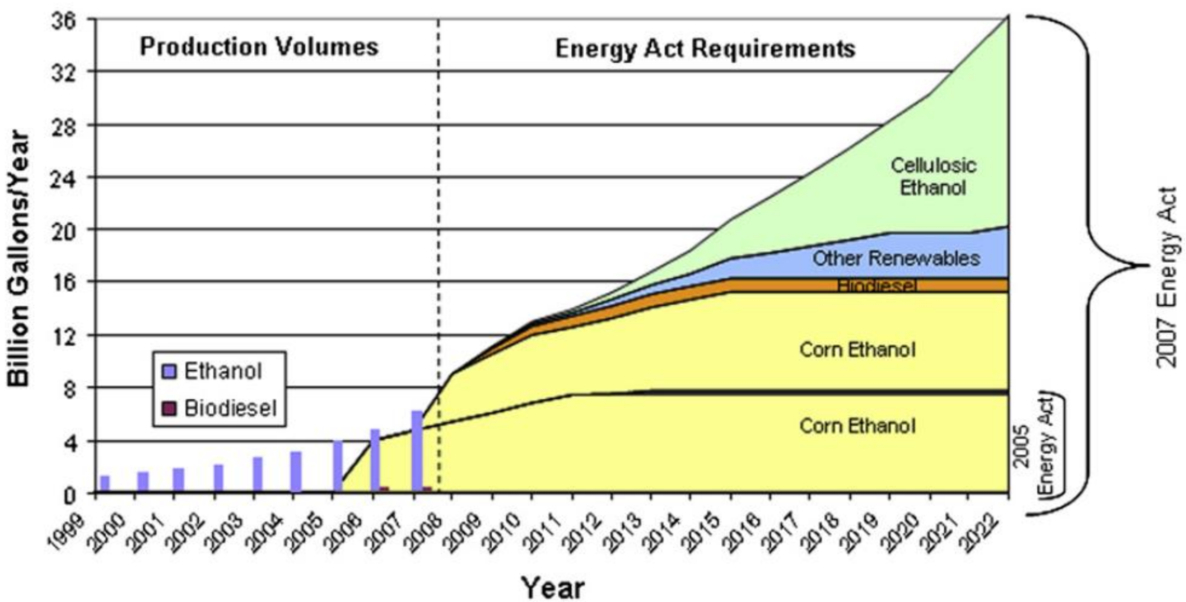


24,918 Btu/gal vs 38,323 Btu/gal

Remarks

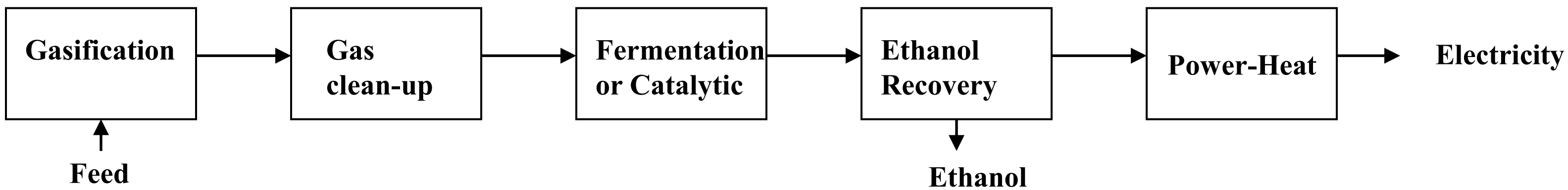
Current ethanol from corn and sugar cane and biodiesel from vegetable oils compete with the food chain.

U.S. Government policies support the production of lignocellulosic based biofuels and the reuse of wastes and new sources (algae)

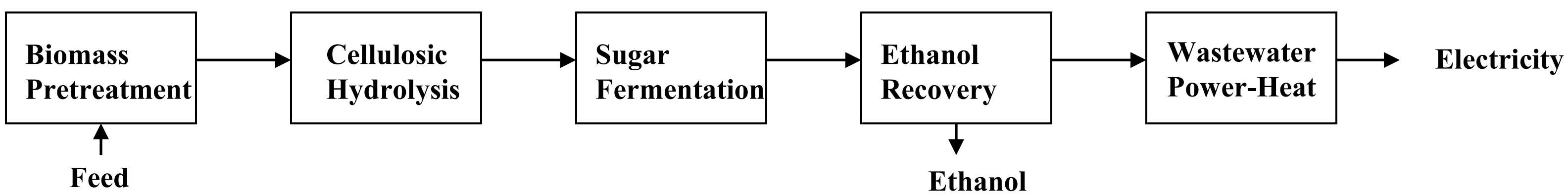


Lignocellulosic Bioethanol

a) Thermochemical Process *(gasification)*



b) Hydrolysis Process *(fermentation)*



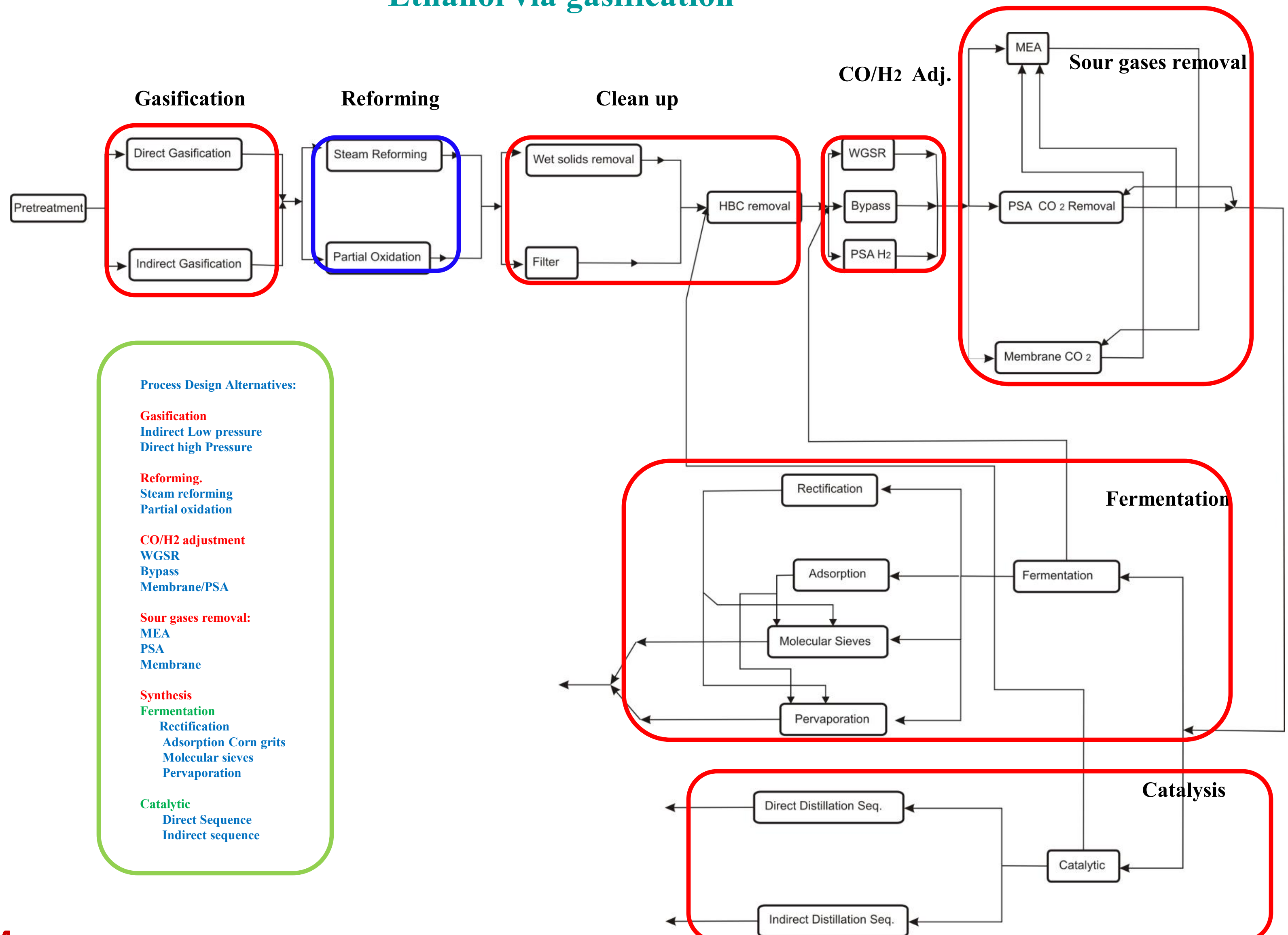
Challenge:

Many alternative flowsheets

Superstructure Thermochemical Bioethanol

Martin, Grossmann (2010)

Ethanol via gasification



Process Design Alternatives:

Gasification
Indirect Low pressure
Direct high Pressure

Reforming.
Steam reforming
Partial oxidation

CO/H2 adjustment
WGSR
Bypass
Membrane/PSA

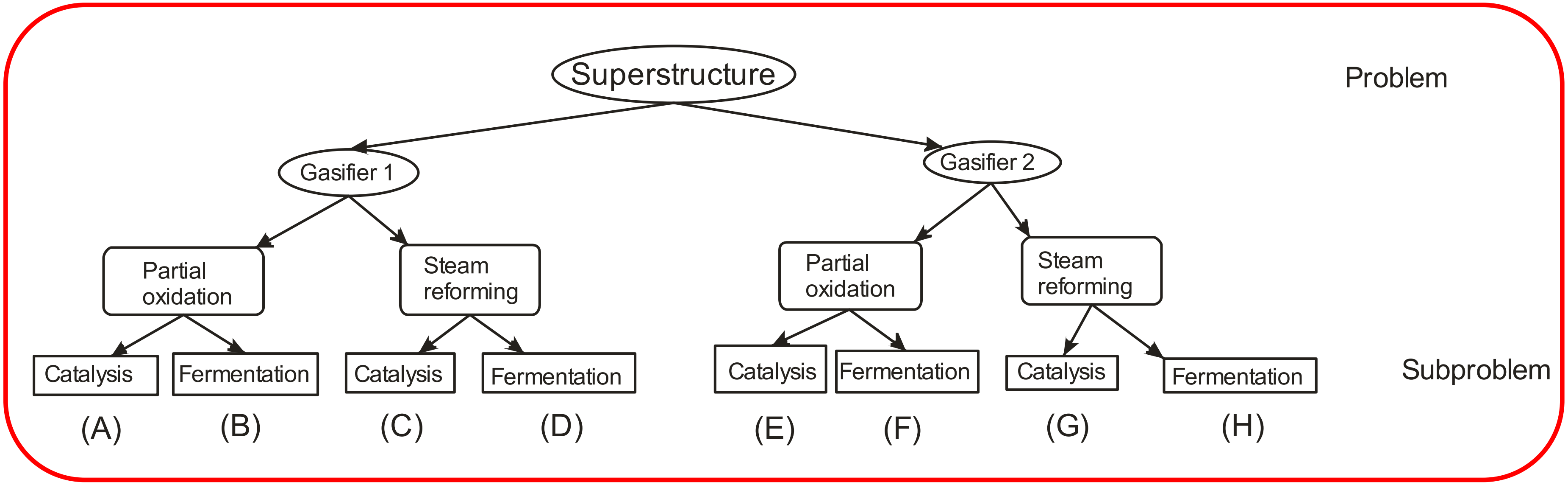
Sour gases removal:
MEA
PSA
Membrane

Synthesis
Fermentation
- Rectification
- Adsorption Corn grits
- Molecular sieves
- Pervaporation

Catalytic
- Direct Sequence
- Indirect sequence

-Martin, M. Grossmann, I. E (2010) Aiche J. Submitted

Solution Strategy Energy Optimization



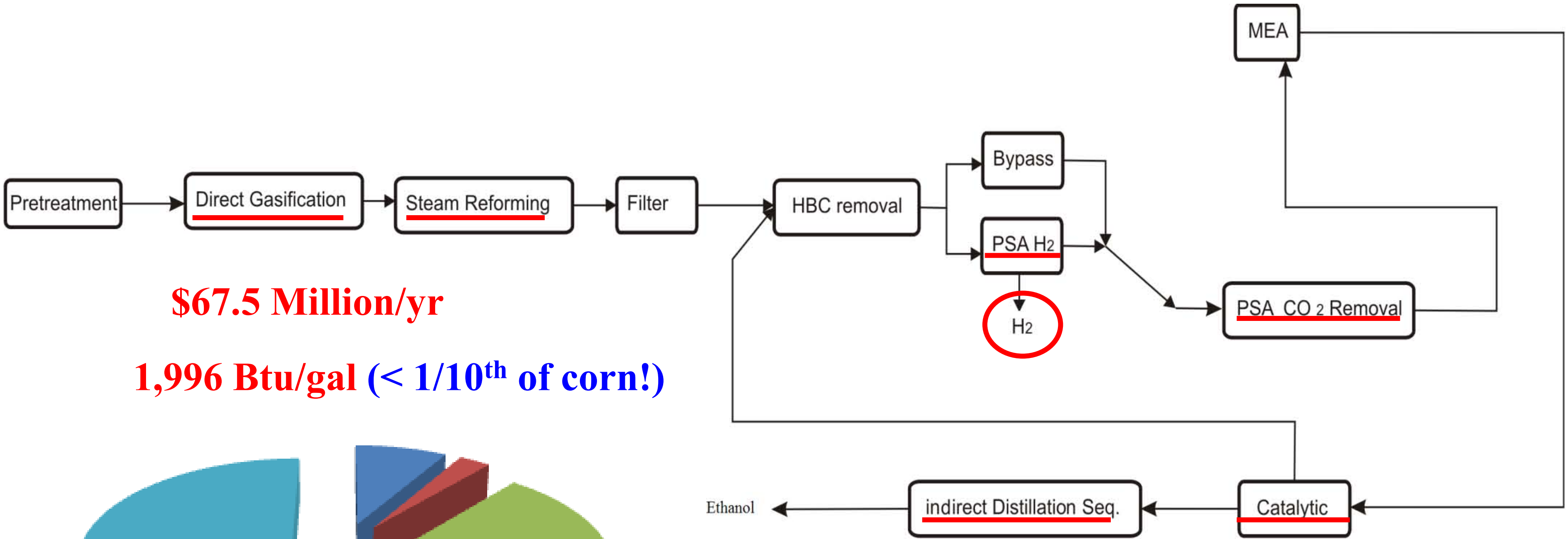
Decomposition of GDP in 8 subproblems

Decision levels: Gasifier
Removal HCs
Reaction of Syn Gas

↓

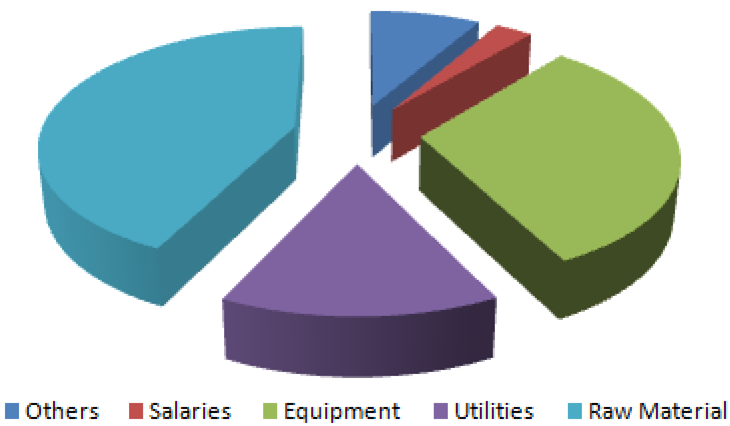
Heat integration and economic evaluation

Optimal Design of Lignocellulosic Ethanol Plant



$67.5 Million/yr

1,996 Btu/gal (< 1/10th of corn!)



Ethanol: $0.81 /gal *(no H_2 credits)*
$ 0.42/gal *(H_2 credits)*

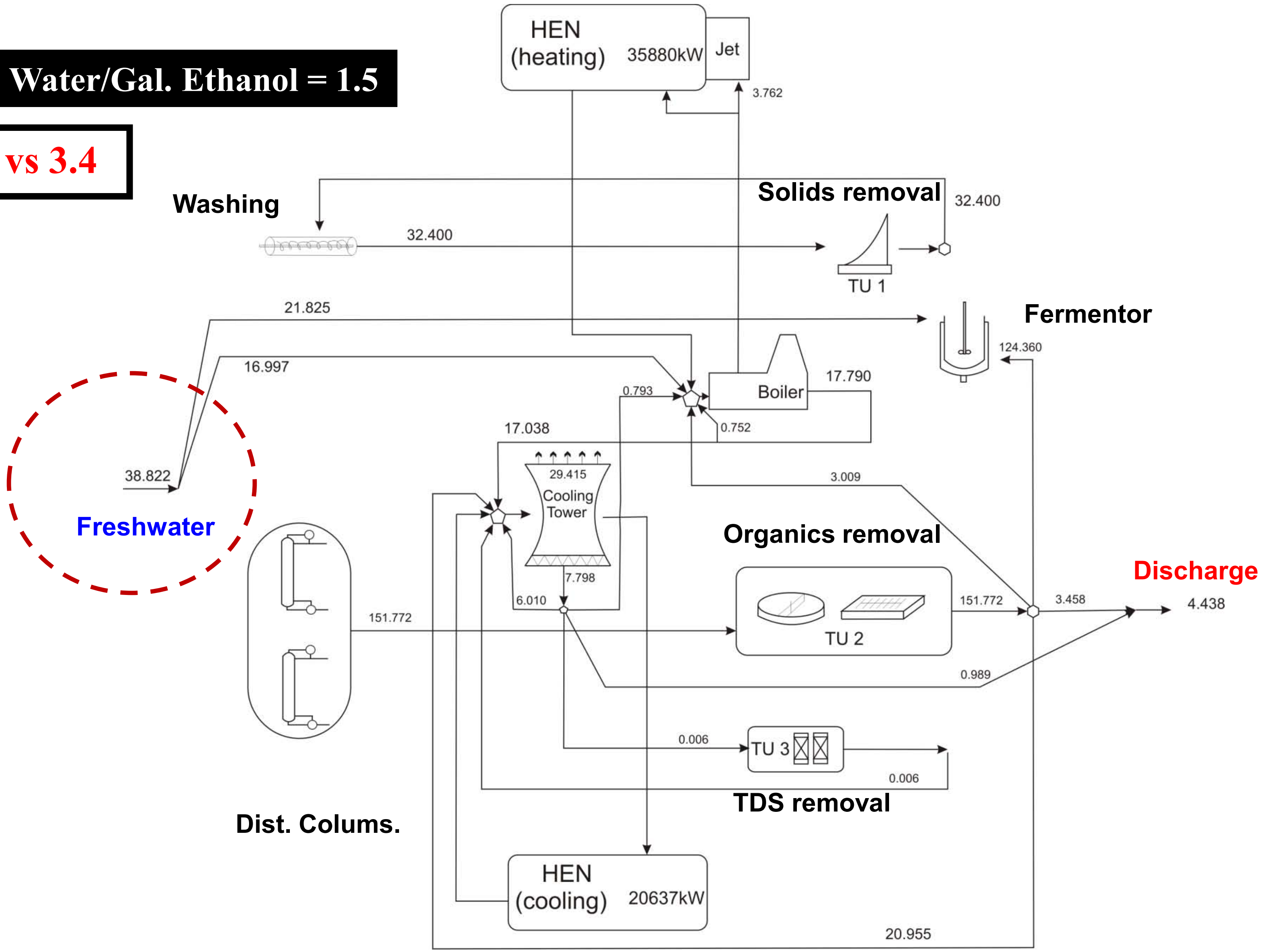
Each NLP subproblem: 7000 eqs., 8000 var
~25 min to solve

Low cost is due to H_2 production

Optimal Water Network: Corn Ethanol

Gal. Water/Gal. Ethanol = 1.5

1.5 vs 3.4

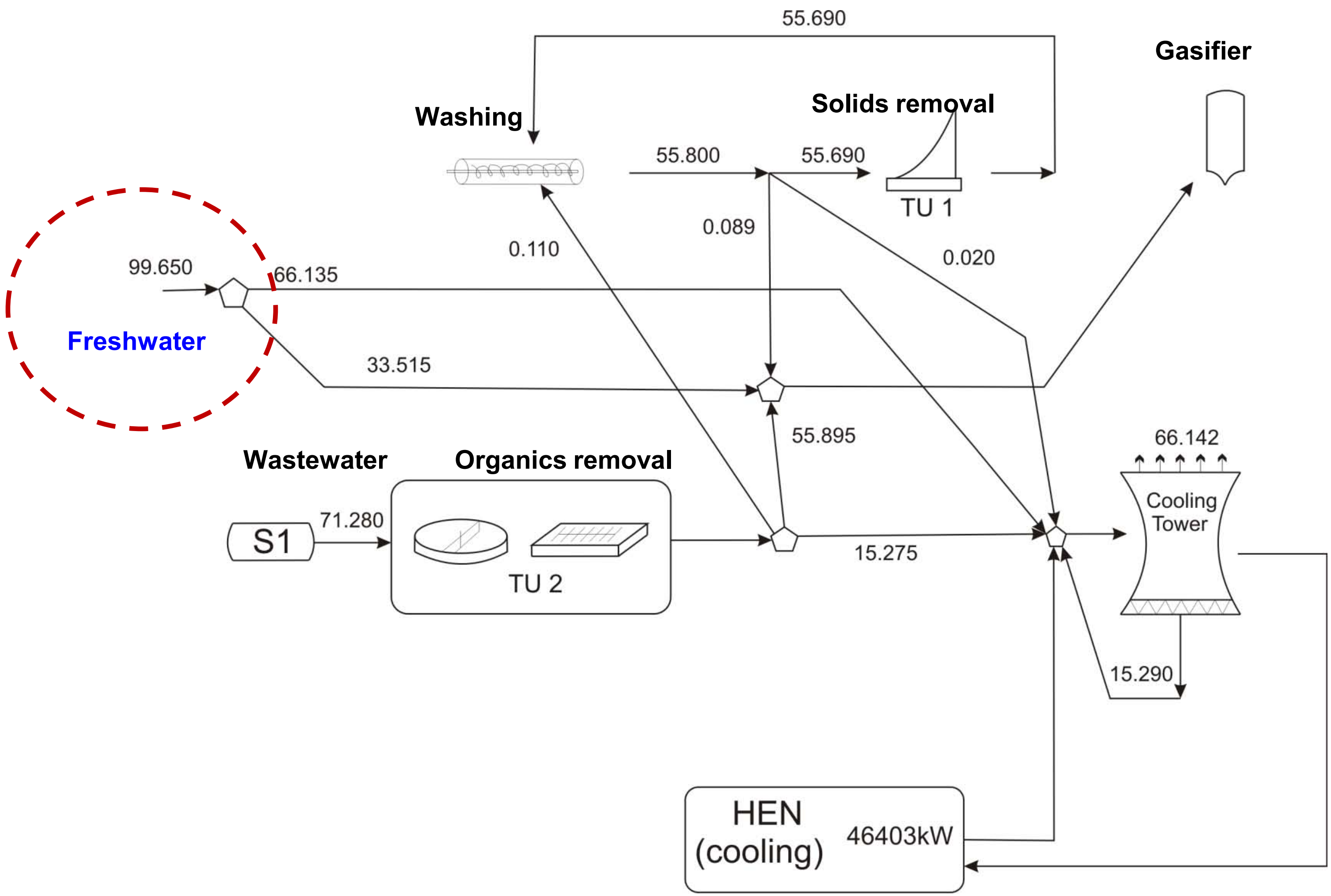


-Ahmetović , E., Martin, M. Grossmann, (2009) I&ECR. **2010,** ***49*****, *7972–7982***

Optimal Water Network: Lignocellulosic Ethanol

Gal. Water/Gal. Ethanol = 4.2

Cellulosic Bioethanol via Gasification



-Martin, M. , Ahmetović, E., Grossmann, I. E (2010) I&ECR ASAP

Table Summary of results [6-10]

	Ethanol (Hydrolysis)	Ethanol (Gasification & Catalysis)	Ethanol (Gasification & Fermentation)	FT-Diesel	Hydrogen	Biodiesel (Cooking)	Biodiesel (Algae)
	A	**C**		**D**	**B**	**F**	**E**
Total investment ($MM)	161	335	260	212	148	17	102
Capacity(MMgal/yr)	60	60	60	60	60*	72	72
Biofuel yield (kg/kg_{wet})	0.28	0.20	0.33	0.24	0.11	0.96	0.48
Production cost ($/gal)	0.80	0.41	0.81	0.72	0.68*	0.70	0.47
Water consumption(gal/gal)	1.66	0.36	1.59	--	--	0.33	0.60
Energy consump. (MJ/gal)	-10.2	-9.5	27.2	-60.0	-3.84*	1.94	1.94
Byproduct	Energy CO_2	Hydrogen Energy CO_2	Hydrogen CO_2	Green Gasoline Energy CO_2	Energy CO_2	Glycerol	Glycerol Fertilizer

(*) kg instead of gal

[6] Martín, M., Grossmann, I.E. (2011) AIChE J. DOI: 10.1002/aic.12544
[7] Martín, M., Grossmann, I.E. Energy optimization of Hydrogen production from biomass. Rev. Submited to Comp. Chem. Eng.
[8] Martín, M., Grossmann, I.E. Energy optimization of lignocellulosic bioethanol production via Hydrolysis to be submitted AIChE J.
[9] Martín, M., Grossmann, I.E. Process optimization of FT- Diesel production from biomass. To be submitted
[10] Martín, M., Grossmann, I.E. Process optimization bioDiesel production from cooking oil and Algae. To be submitted

Design and Planning under Uncertainty

Goal: robustness in decisions

Design of Responsive Supply Chains

Uncertain demands

Maximize NPV/Minimize responsiveness

Chance constrained MINLP

Design and Planning of Offshore Oilfields

ExxonMobil

Uncertain fields size, deliverability, water

Maximize expected flexibility/Minimize Cost

Multi-stage programming MINLP

Optimal Design of Responsive Process Supply Chains

Fengqi You

Objective: design supply chains under responsive and economic criteria with consideration of inventory management and demand uncertainty

Problem Statement

Suppliers

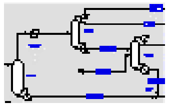
Plants

Where?
What?
When?

DCs

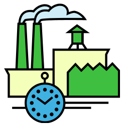
Customers

Production Network

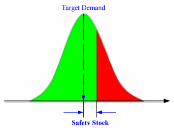


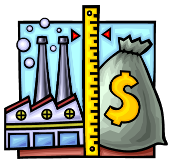
Costs and prices

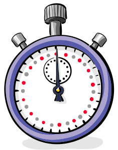
Production and transportation time



Demand information

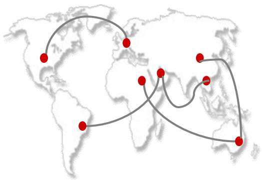






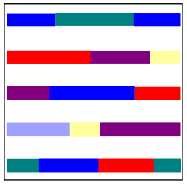
Max: Net present value

Max: Responsiveness



Network Structure

Operational Plan



Production Schedule

Production Network of Polystyrene Resins

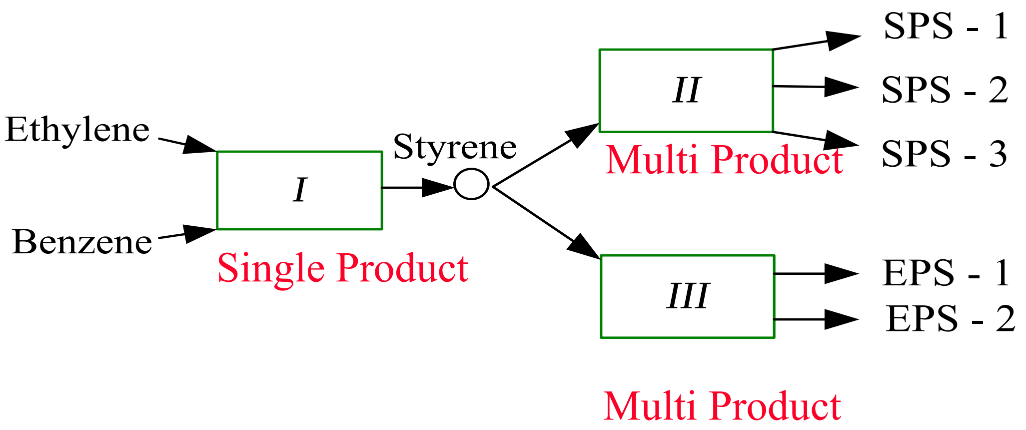
Three types of plants:

Plant *I*: *Ethylene + Benzene* ⟶ *Styrene* (1 products)

Plant *II*: *Styrene* ⟶ *Solid Polystyrene (SPS)* (3 products)

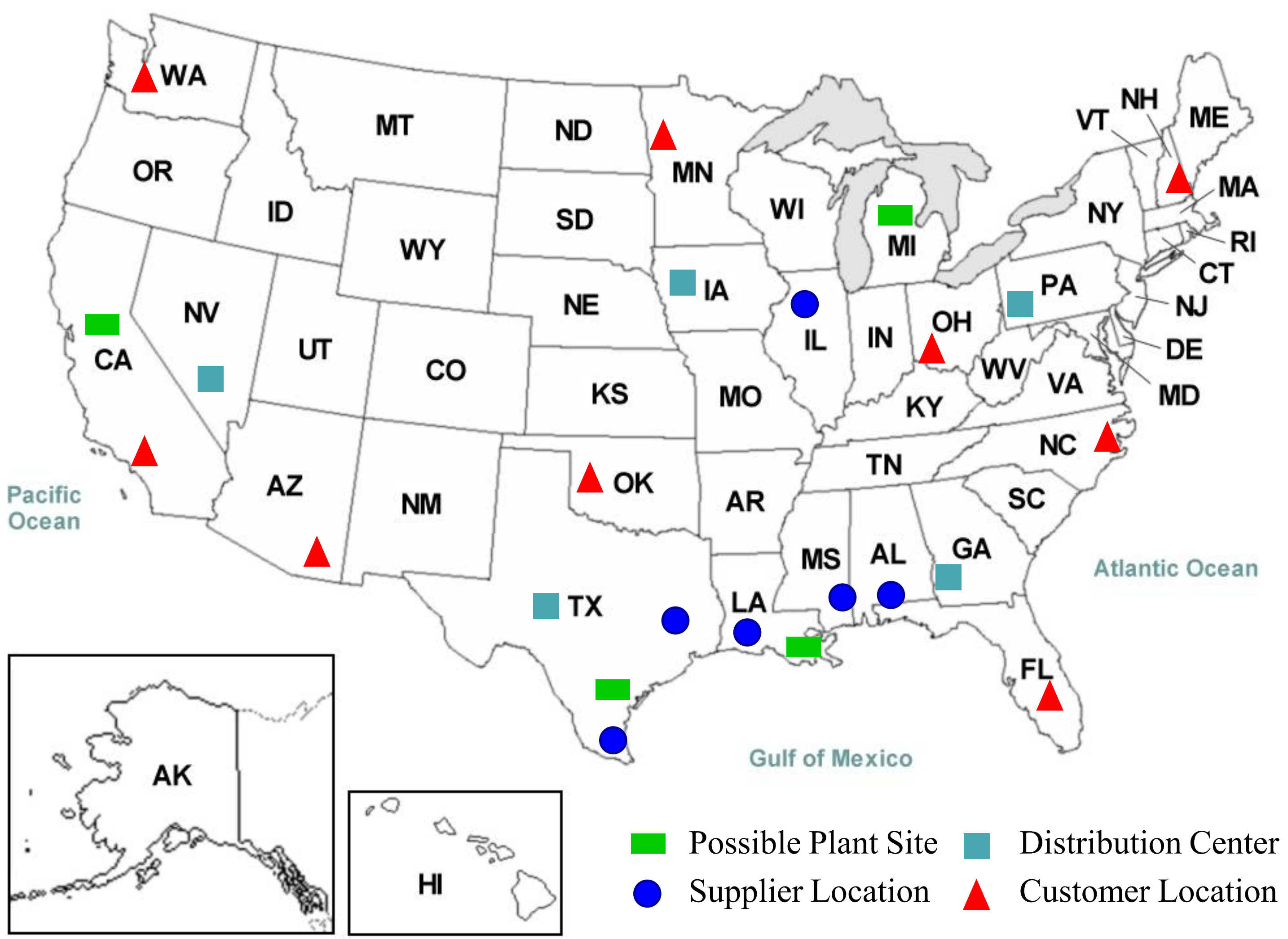
Plant *III*: *Styrene* ⟶ *Expandable Polystyrene (EPS)* (2 products)

Basic Production Network

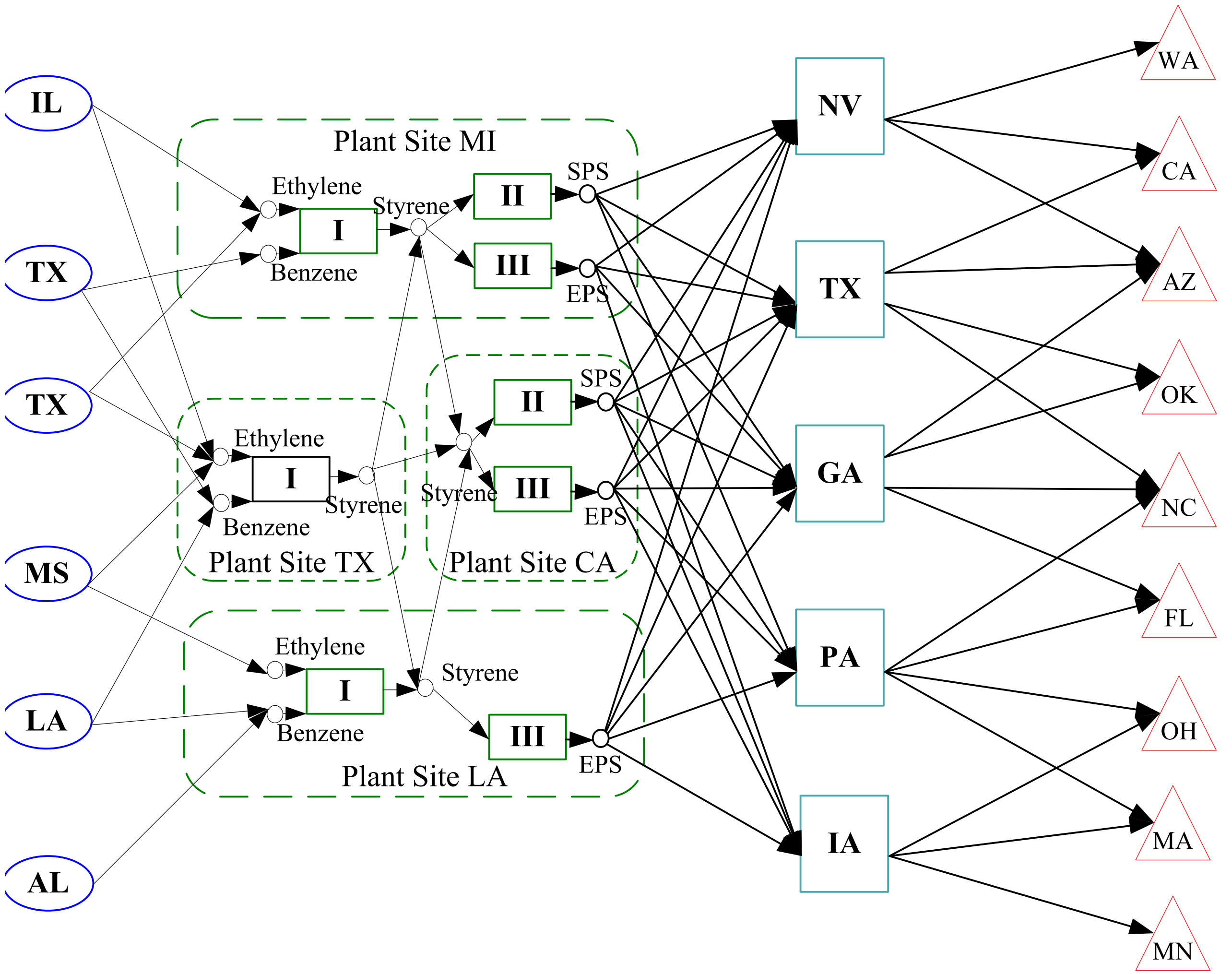


Source: Data Courtesy Nova Chemical Inc. *http://www.novachem.com/*

Location Map

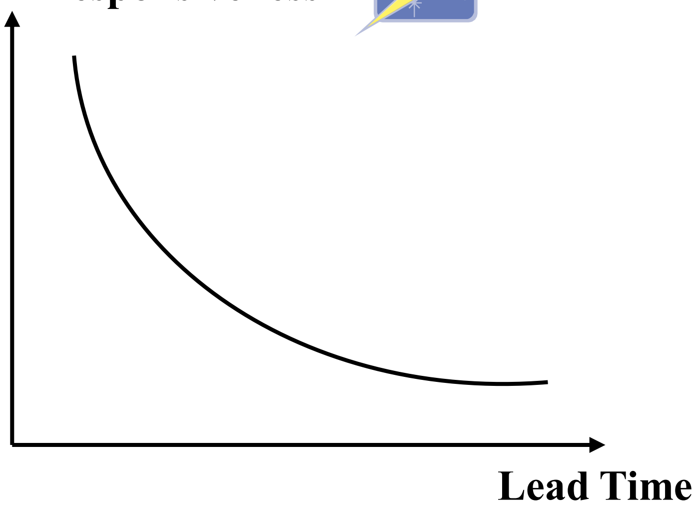


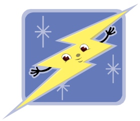
Potential Network Superstructure

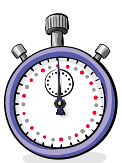


Responsiveness - Lead Time

- Lead Time: The time of a supply chain network to respond to customer demands and preferences in the worst case



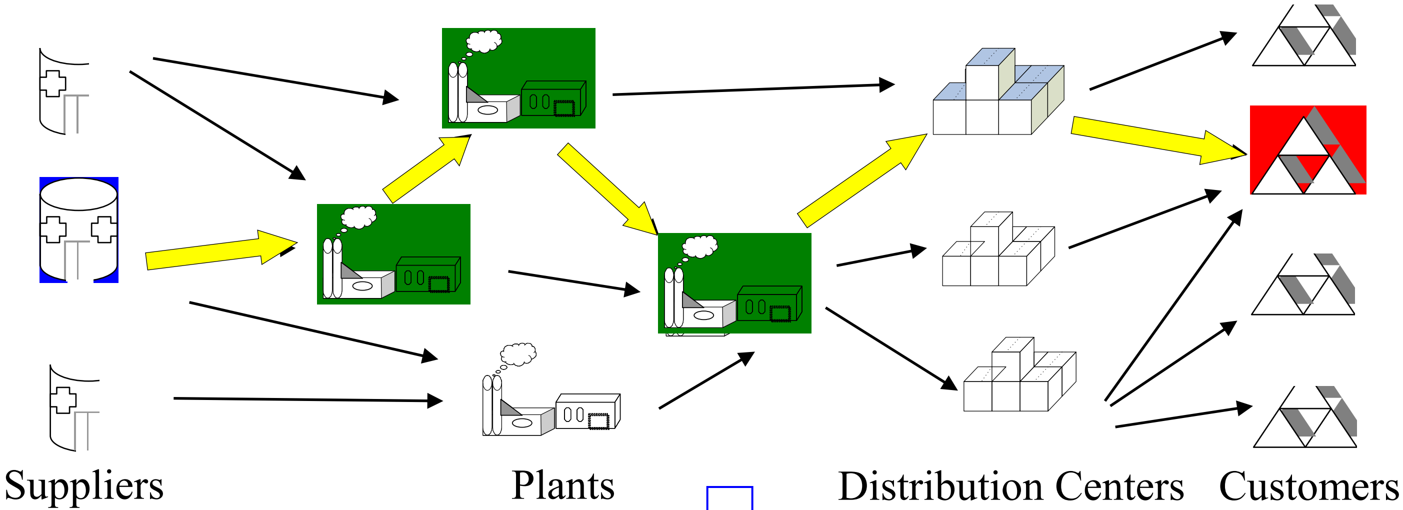


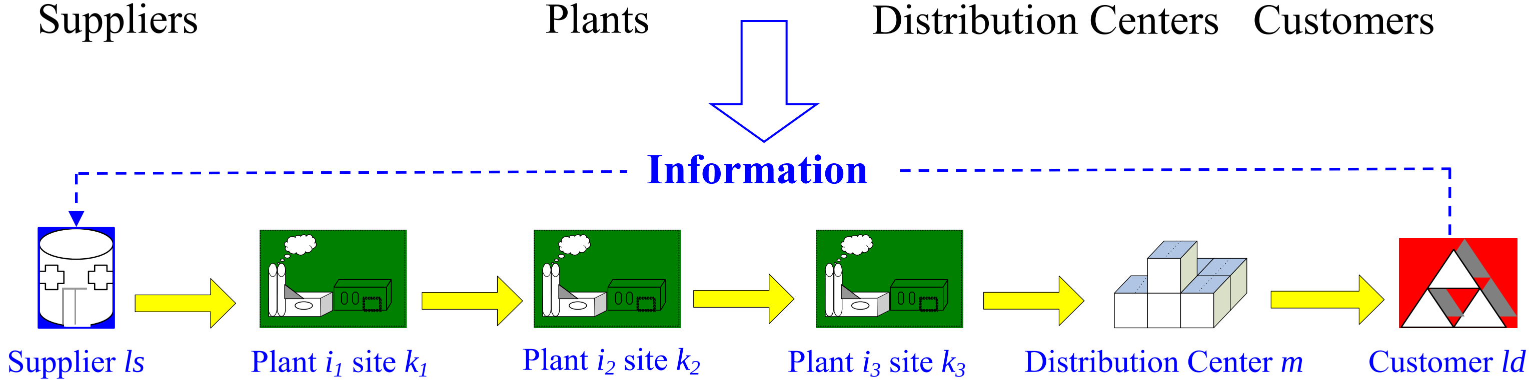


Lead Time is a measure of responsiveness in SCs

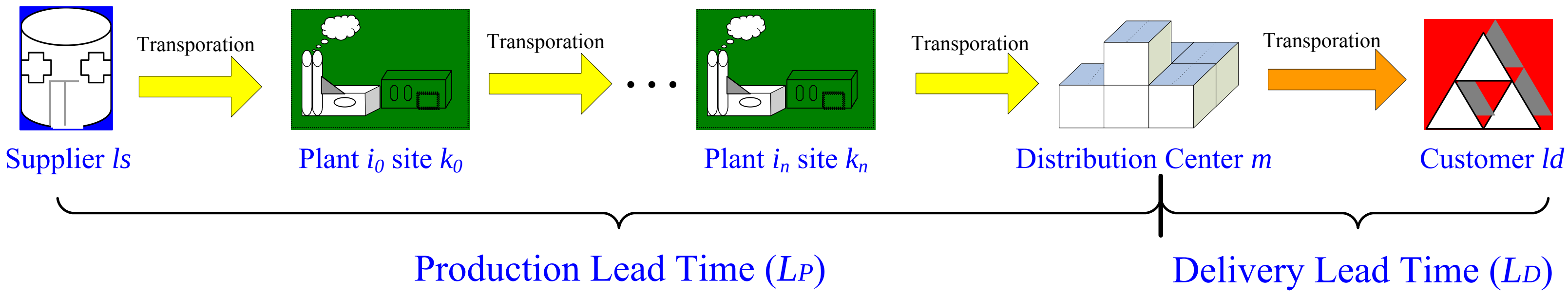
Lead Time for A Linear Supply Chain

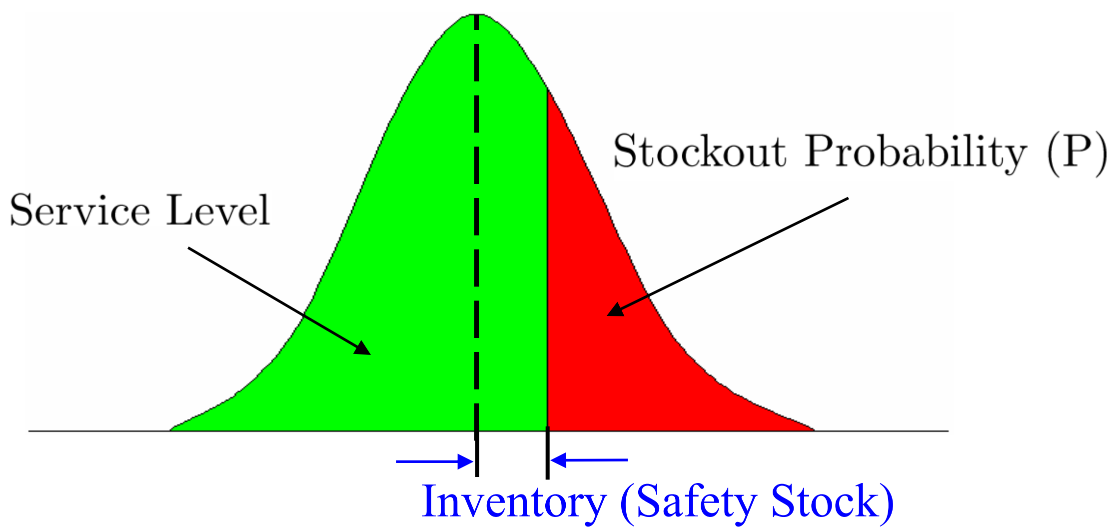
- A supply chain network = ∑Linear supply chains
  - Assume information transfer instantaneously





Lead Time under Demand Uncertainty

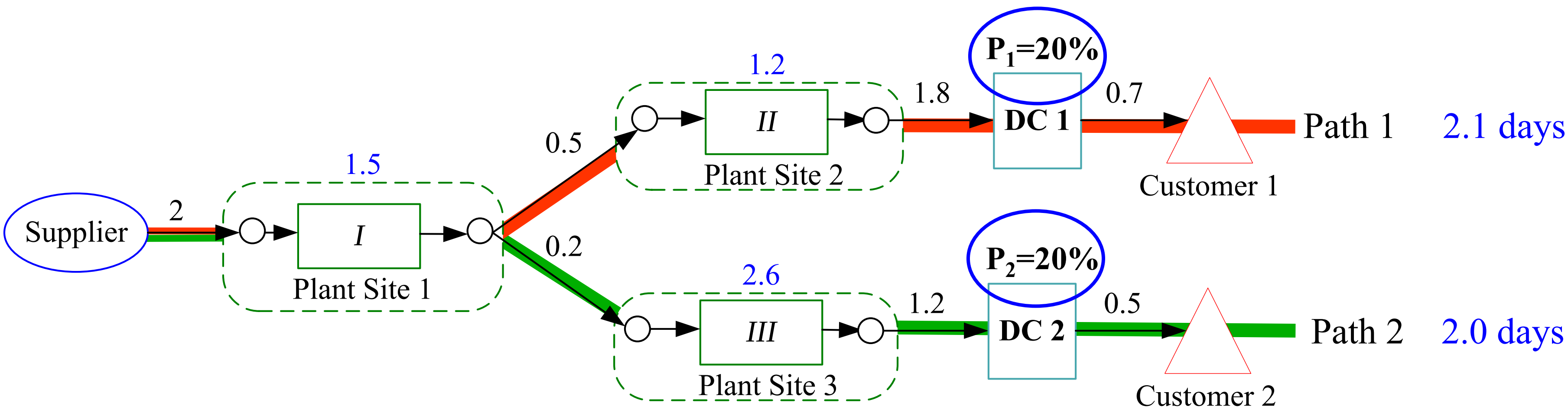




$$\text{Expected Lead Time} = L_D + \text{P(Stockout)} \cdot L_P$$

Expected Lead Time of SCN

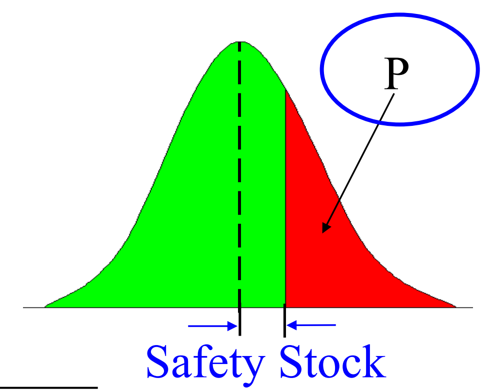
- Expected Lead time of a supply chain network (uncertain demand)
  - The longest expected lead time for all the paths in the network (***worst case***)
  - Example: A simple SC with all process are dedicated



For Path 1: $(2 + 1.5 + 0.5 + 1.2 + 1.8) \times \mathbf{20\%} + 0.7 = 2.1$ days

For Path 2: $(2 + 1.5 + 0.2 + 2.6 + 1.2) \times \mathbf{20\%} + 0.5 = 2.0$ days

Expected Lead Time = max $\{2.1, 2.0\} = 2.1$ days

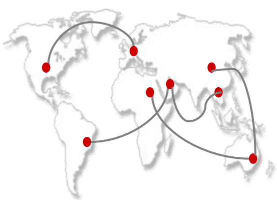


Bi-criterion Multiperiod MINLP Formulation

Choose Discrete (0-1), continuous variables

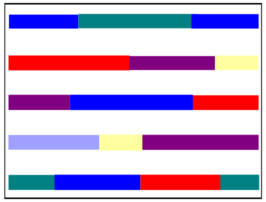
- Objective Function:
  - Max: Net Present Value
  - Min: Expected Lead time

  } Bi-criterion
- Constraints:
  - Network structure constraints

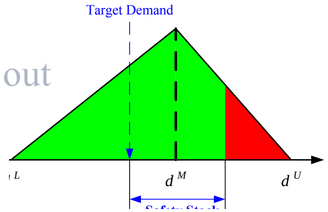


    - Suppliers – plant sites Relationship
    - Plant sites – Distribution Center
    - Input and output relationship of a plant
    - Distribution Center – Customers
    - Cost constraint
  - Operation planning constraints

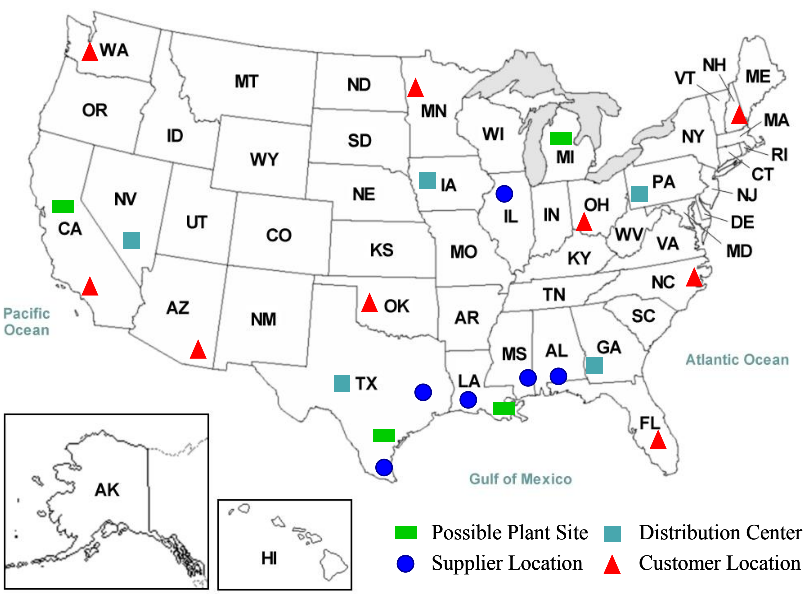
    - Production constraint
    - Capacity constraint
    - Mass balance constraint
    - Demand constraint
    - Upper bound constraint
  - Cyclic scheduling constraints

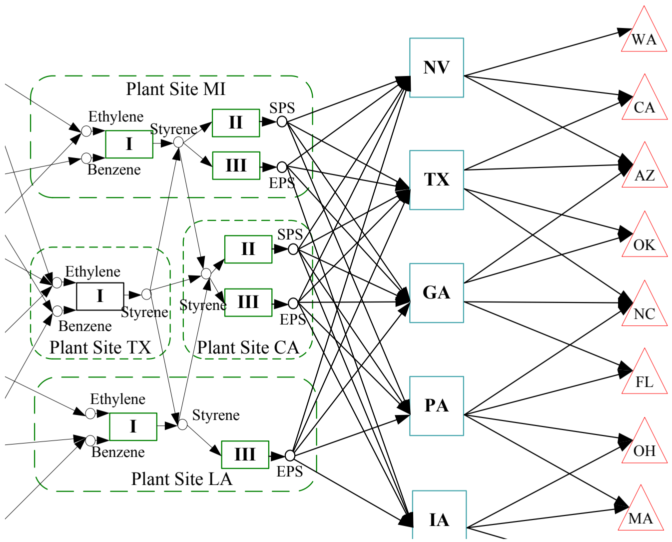


    - Assignment constraint
    - Sequence constraint
    - Demand constraint
    - Production constraint
    - Cost constraint
  - Probabilistic constraints
    - Chance constraint for stock out (reformulations)



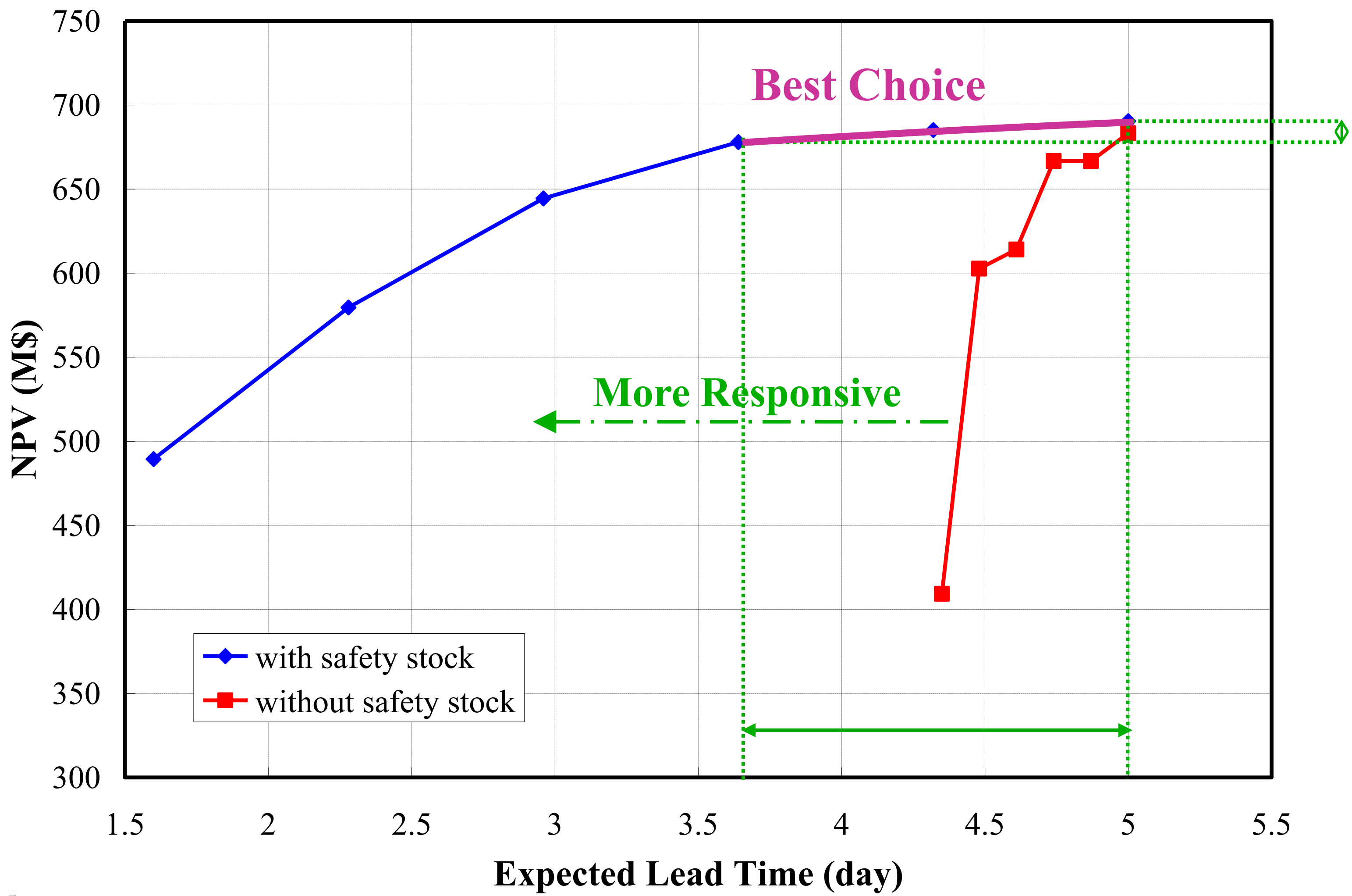
Case Study



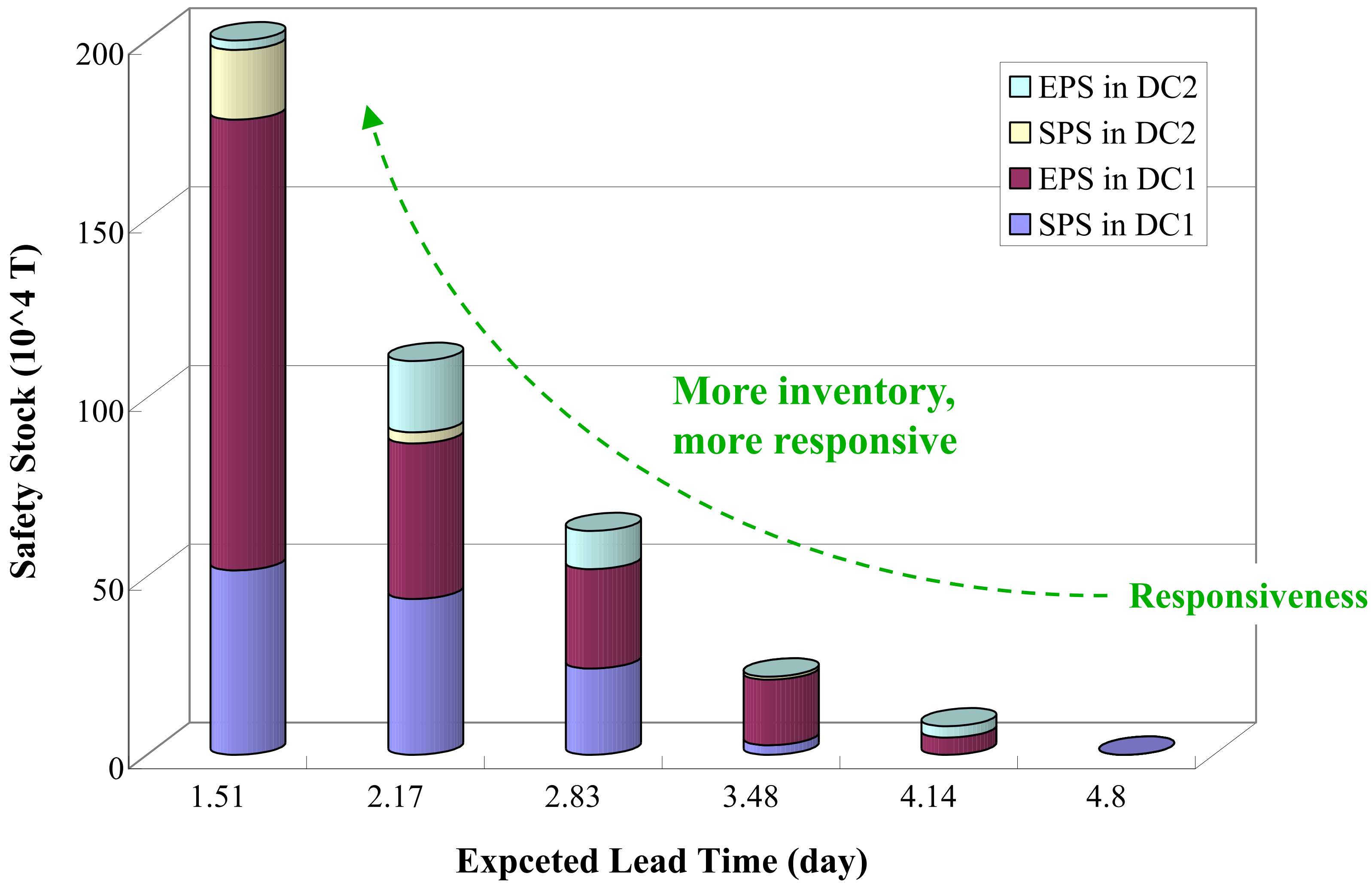


- Problem Size:
  - # of Discrete Variables: 215
  - # of Continuous Variables: 8126
  - # of Constraints: 14617
- Solution Time:
  - Solver: GAMS/BARON
  - Direct Solution: > 2 weeks
  - Proposed Algorithm: ~ 4 hours

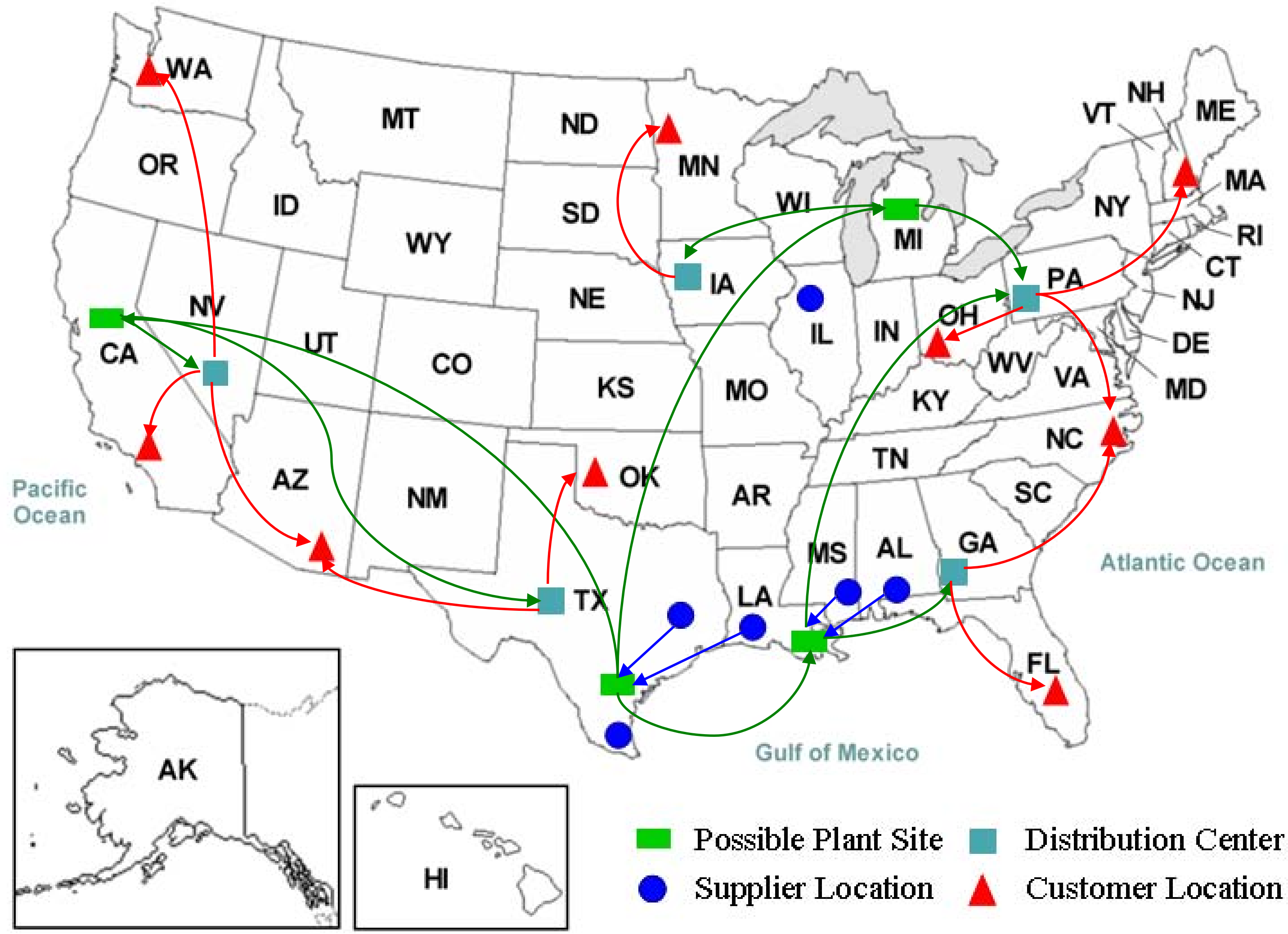
Pareto Curves – with and without safety stock



Safety Stock Levels - Expected Lead Time



Network Structure at Location Map



Optimal Development Planning under Uncertainty

➢Offshore oilfield having several reservoirs under uncertainty

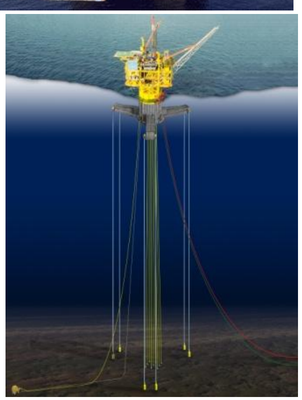
Tarhan, Grossmann (2009)

➢Maximize the expected net present value (ENPV) of the project

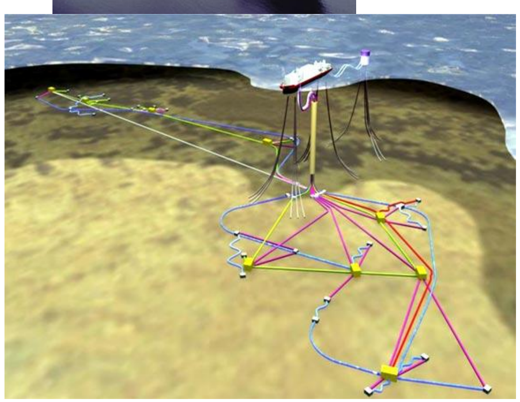
Decisions:

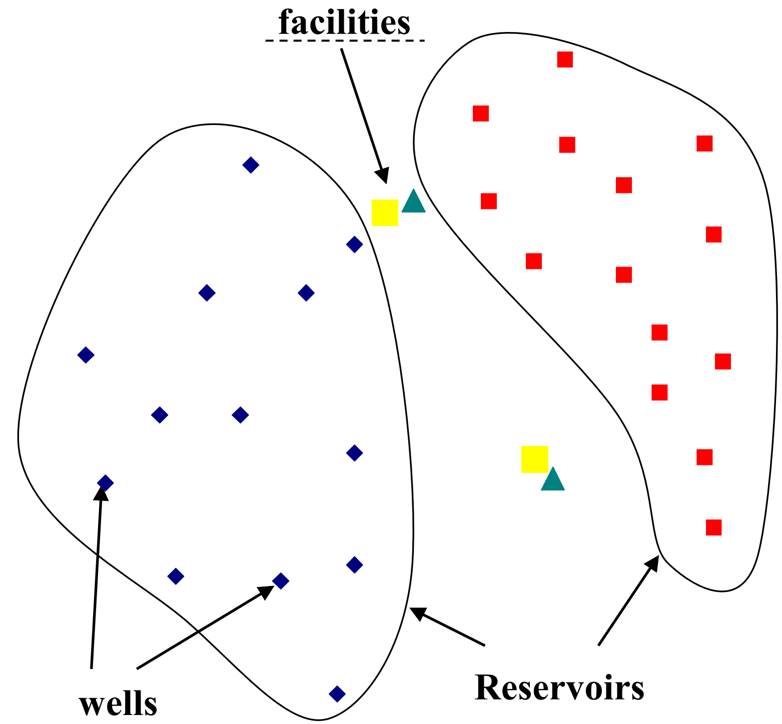
- ➢ Number and capacity of TLP/FPSO facilities
- ➢ Installation schedule for facilities
- ➢ Number of sub-sea/TLP wells to drill
- ➢ Oil production profile over time

TLP



FPSO





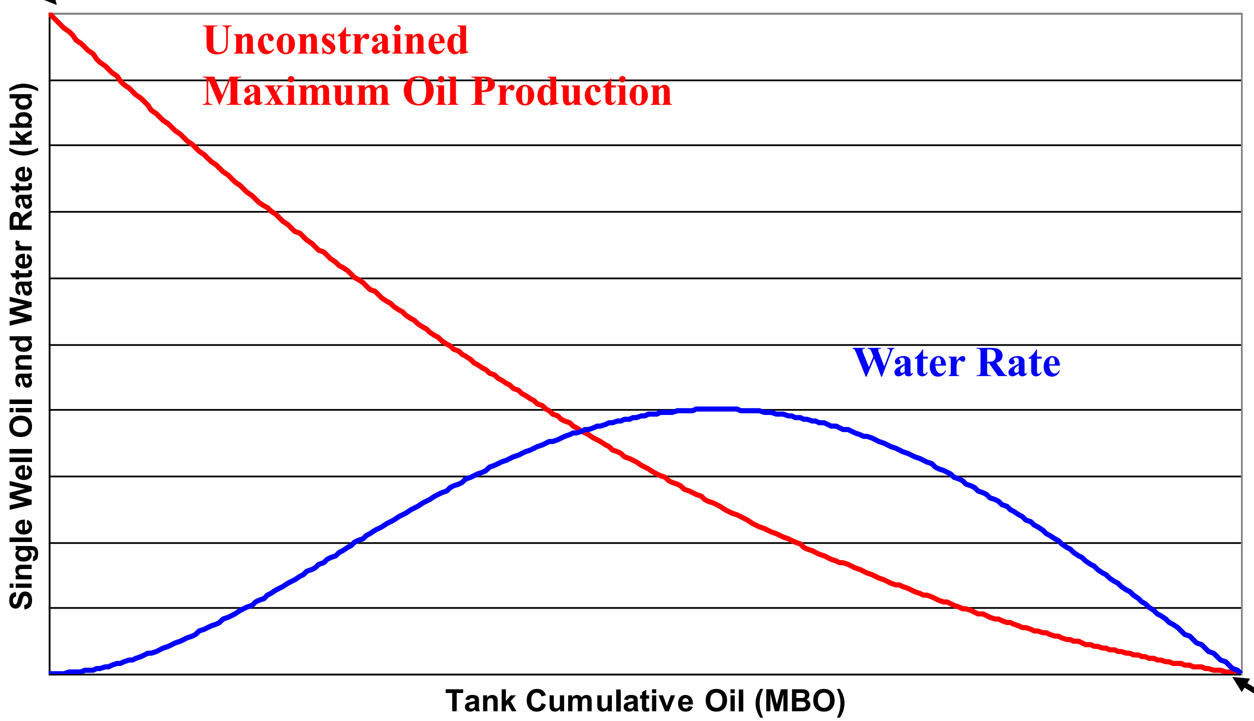
Uncertainty:

- ➢**Initial productivity per well**
- ➢**Size of reservoirs**
- ➢**Water breakthrough time for reservoirs**

Non-linear Reservoir Model

Assumption: All wells in the same reservoir are identical.

Initial oil production

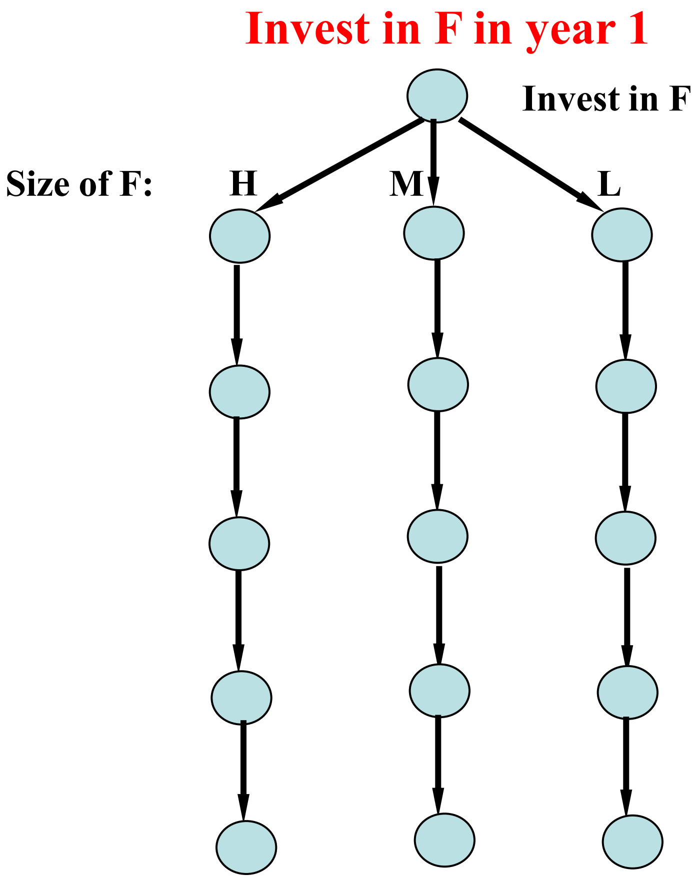


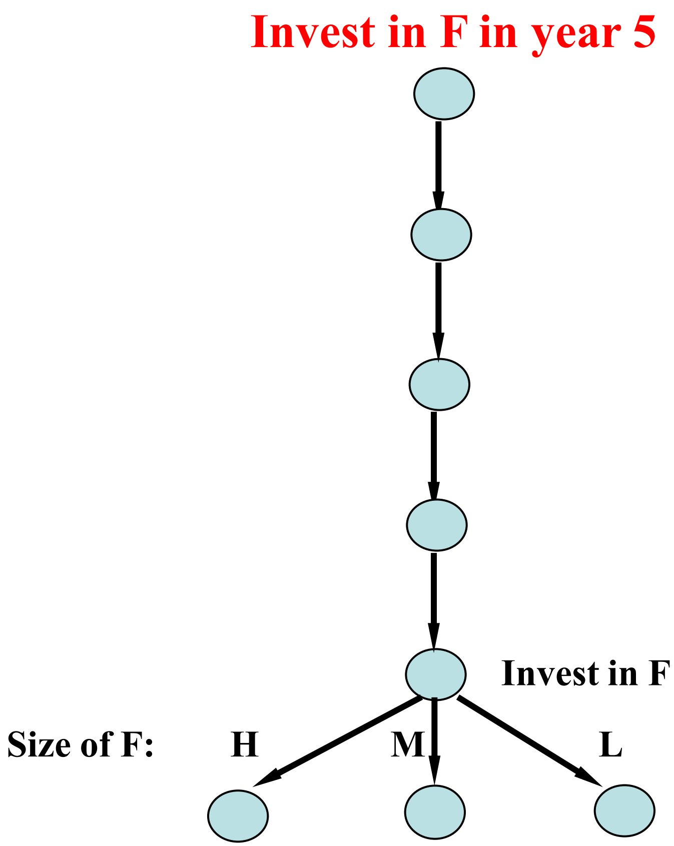
Size of the reservoir

Uncertainty is represented by discrete distributions functions

Decision Dependent Scenario Trees

Assumption: Uncertainty in a field resolved as soon as WP installed at field



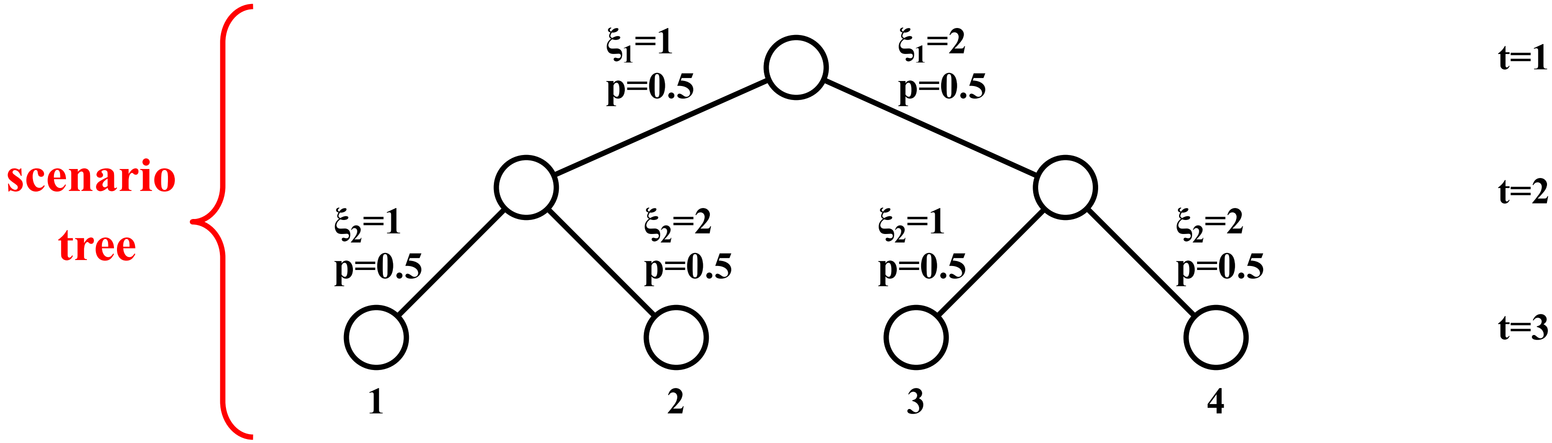


Scenario tree

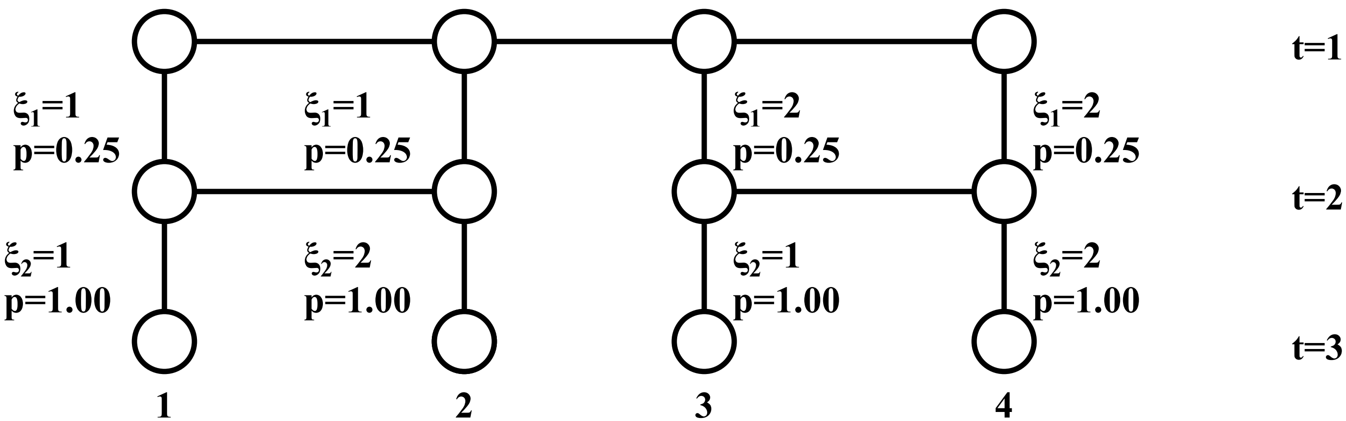
Not unique: Depends on timing of investment at uncertain fields

Central to defining a Stochastic Programming Model

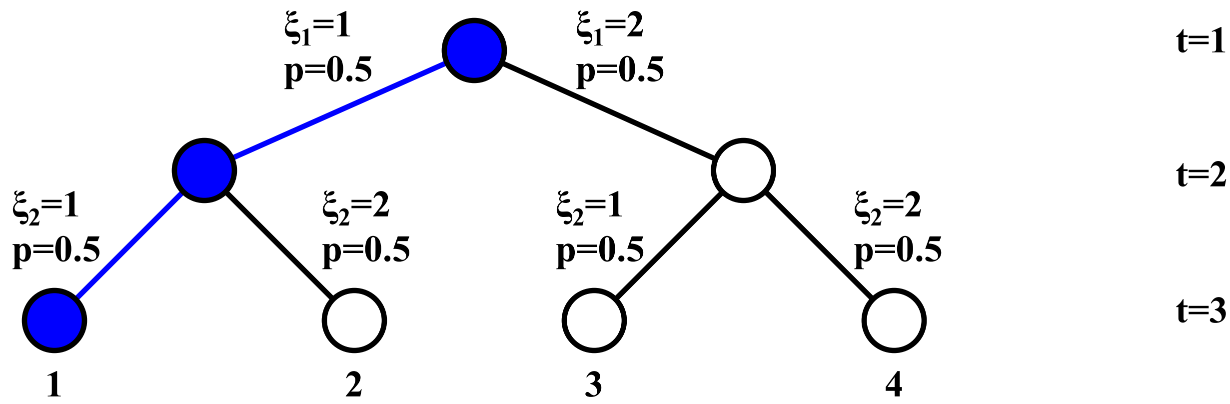
Stochastic Programming



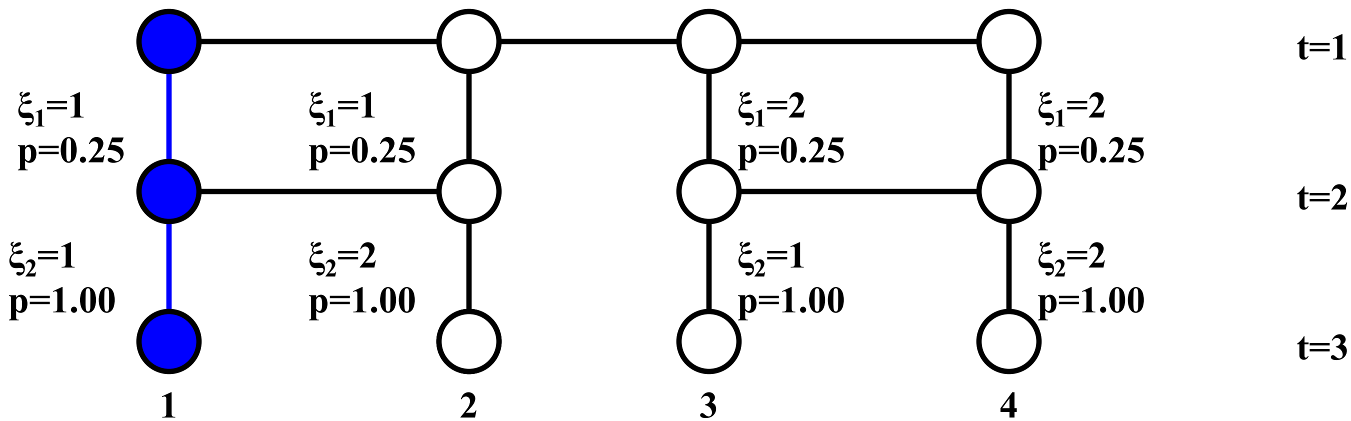
Alternative and equivalent scenario tree structure (Ruszczynski, 1997):



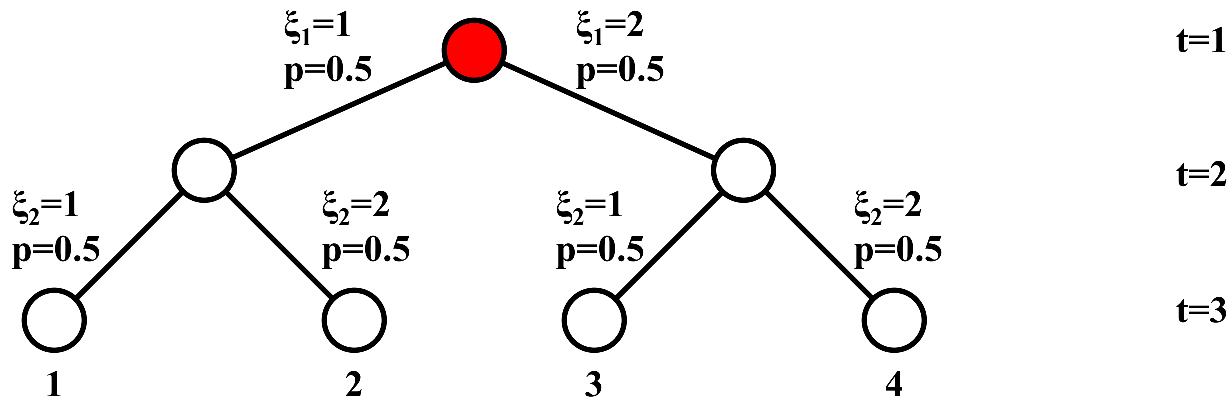
Stochastic Programming



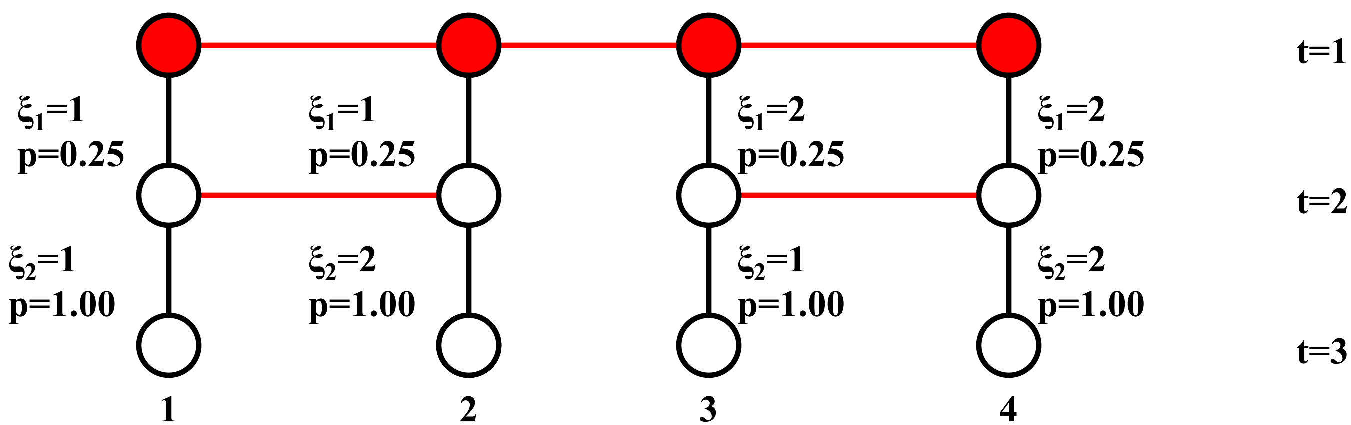
Each scenario is represented by a set of unique nodes



Stochastic Programming

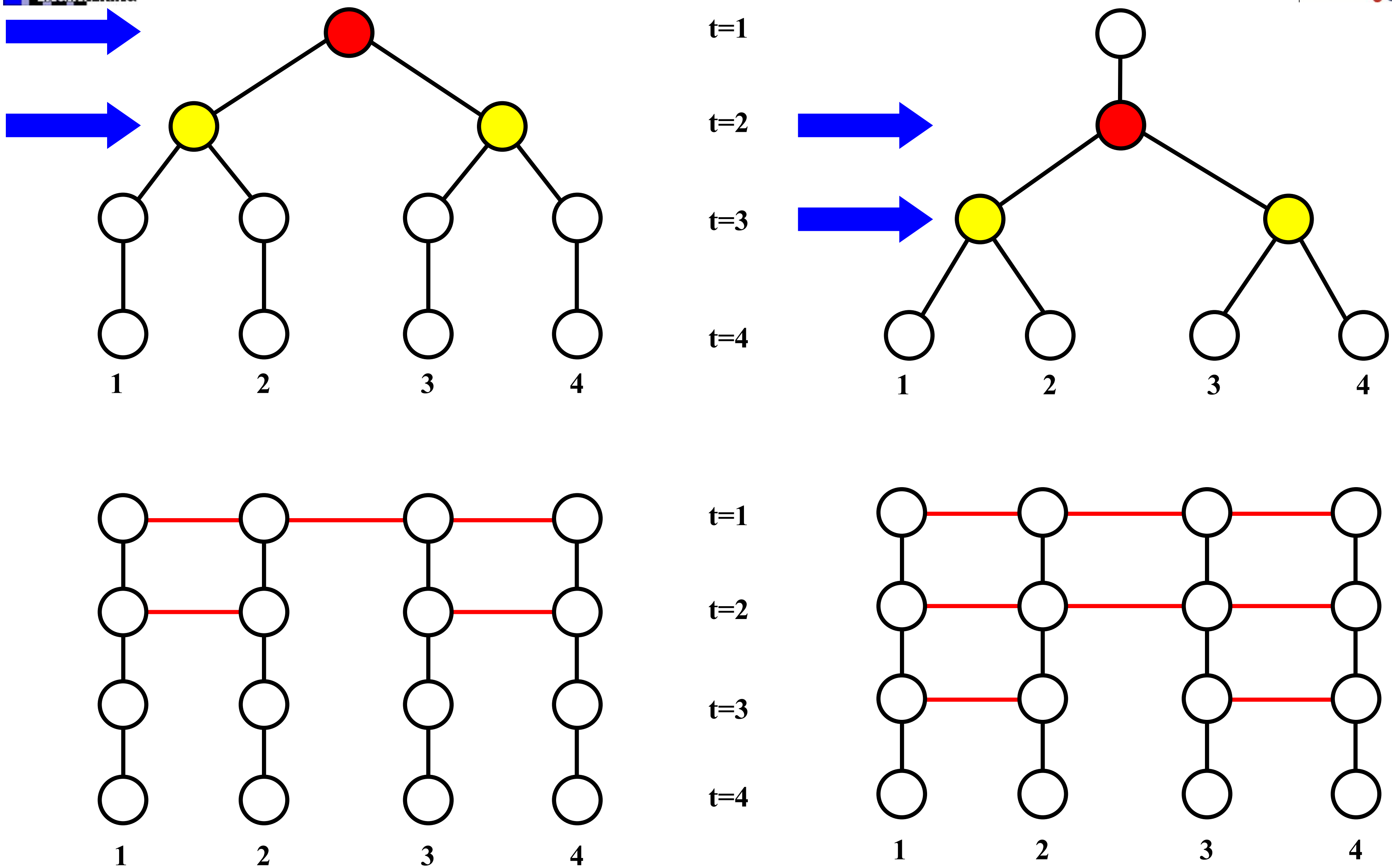


Nodes have **same amount of information** $\equiv$ **Nodes are** **indistinguishable**



Non-anticipativity constraints

Representation of Decision-Dependence Using Scenario Tree



Multi-stage Stochastic Nonconvex MINLP

Maximize.. **Probability weighted average of NPV over uncertainty scenarios**

subject to

- **Equations about economics of the model**
- **Surface constraints**
- **Non-linear equations related to reservoir performance**
- **Logic constraints relating decisions**

  if there is a TLP available, a TLP well can be drilled

Every scenario, time period

- **Non-anticipativity constraints**

Non-anticipativity prevents a decision being taken now from using information that will only become available in the future

Disjunctions (conditional constraints)

Every pair scenarios, time period

Problem size MINLP increases exponentially with number of time periods and scenarios ⇒	**Decomposition algorithm:** ***Lagrangean relaxation & Branch and Bound***

MILP Branch and cut: Colvin, Maravelias (2008)

Formulation of Lagrangean dual

Relaxation

- **Relax disjunctions, logic constraints**
- **Penalty for equality constraints**

$^{b}\lambda_{uf}^{s,s'}, {}^{y}\lambda^{s,s'}, {}^{d}\lambda^{s,s'}$: Lagrange Multipliers

$$\text{Max} \sum_s p^s \left[\sum_t \left(c_{1t} q_t^s + c_{2t} d_t^s + c_{3t} y_t^s + \sum_{uf} c_{4t,uf} b_{uf,t}^s \right) \right]$$
$$+ \sum_{(s,s')} \left[\sum_{uf} {}^{b}\lambda_{uf}^{s,s'} \left(b_{uf,1}^s - b_{uf,1}^{s'} \right) + {}^{y}\lambda^{s,s'} \left(y_1^s - y_1^{s'} \right) + {}^{d}\lambda^{s,s'} \left(d_1^s - d_1^{s'} \right) \right]$$

$$\sum_{\tau=1}^{t} \left(A_\tau^s q_\tau^s + B_\tau^s d_\tau^s + C_\tau^s y_\tau^s + \sum_{uf} D_{uf,\tau}^s b_{uf,\tau}^s \right) \le a_t^s \quad \forall (t,s)$$

$$\begin{bmatrix} & Z_t^{s,s'} & & \\ q_t^s & = & q_t^{s'} & \\ d_{t+1}^s & = & d_{t+1}^{s'} & \\ y_{t+1}^s & = & y_{t+1}^{s'} & \\ b_{uf,t+1}^s & = & b_{uf,t+1}^{s'} & \forall uf \end{bmatrix} \vee \left[\neg Z_t^{s,s'} \right] \quad \forall (t,s,s')$$

$$Z_t^{s,s'} \Leftrightarrow \bigwedge_{uf \in \mathcal{D}(s,s')} \left[\bigwedge_{\tau=1}^{t} \left(\neg b_{uf,\tau}^s \right) \right] \quad \forall (t,s,s')$$

$$b_{uf,1}^s = b_{uf,1}^{s'} \quad \forall (uf,s,s')$$
$$d_1^s = d_1^{s'} \quad \forall (s,s')$$
$$y_1^s = y_1^{s'} \quad \forall (s,s')$$

One Reservoir Example

Optimize the planning decisions for an oilfield having single reservoir for 10 years.

Decisions:

Number, capacity and installation schedule of FPSO/TLP facilities

Number and drilling schedule of sub-sea/TLP wells

Oil production profile over time

Uncertain Parameters (Discrete Values)	Scenarios							
	1	2	3	4	5	6	7	8
Initial Productivity per well (kbd)	10	10	20	20	10	10	20	20
Reservoir Size (Mbbl)	300	300	300	300	1500	1500	1500	1500
Water Breakthrough Time Parameter	5	2	5	2	5	2	5	2

Wells are drilled in groups of 3.

Maximum number of 12 sub-sea wells per year can be drilled.

Maximum of 6 TLP wells per year per TLP facility can be drilled.

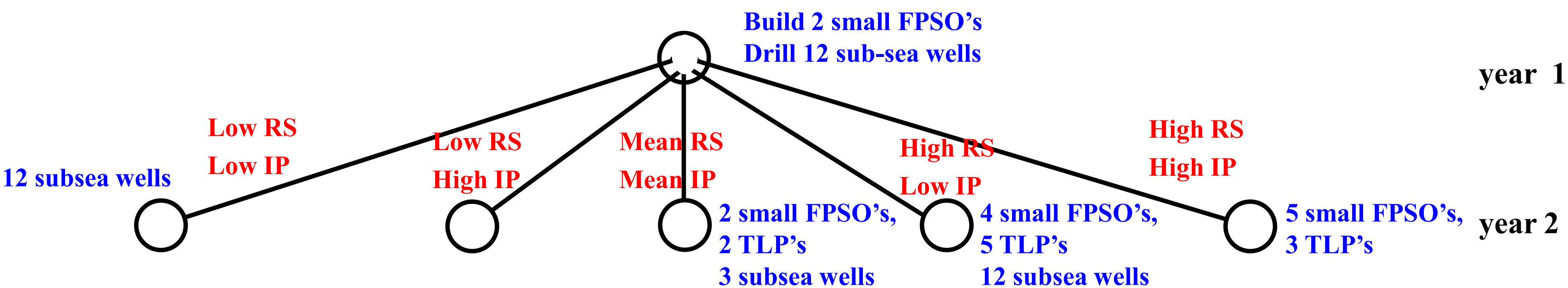
Maximum of 30 TLP wells can be connected to a TLP facility.

Construction Lead Time (years)	Wells		Facilities		
	TLP	Sub-sea	TLP	Small FPSO	Large FPSO
	1	1	1	2	4

Multistage Stochastic Programming Approach

RS: Reservoir size
IP: Initial Productivity
BP: Breakthrough Parameter

E[NPV] = $4.92 x 10^9$



Solution proposes building 2 small FPSO's in the first year and then add new facilities / drill wells (recourse action) depending on the positive or negative outcomes.

Multistage Stochastic Programming Approach

RS: Reservoir size
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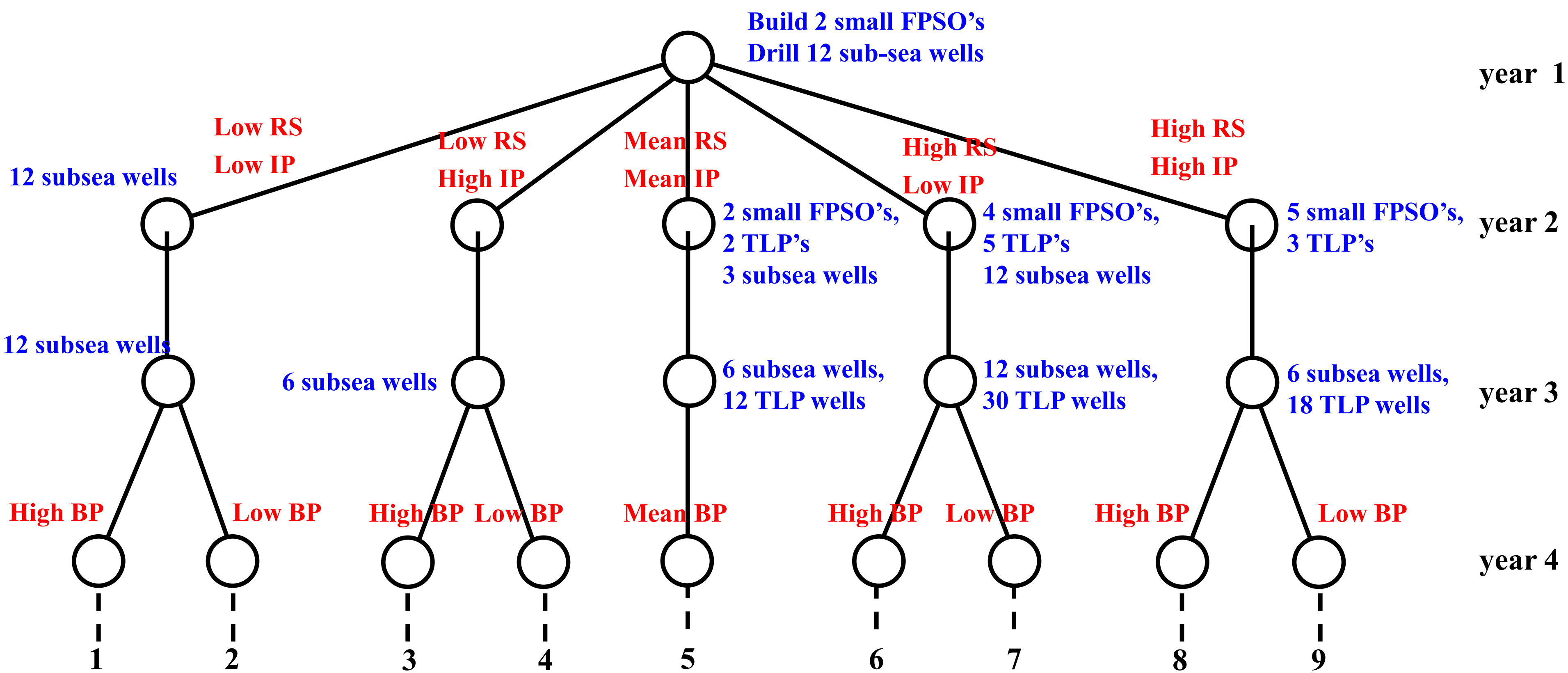
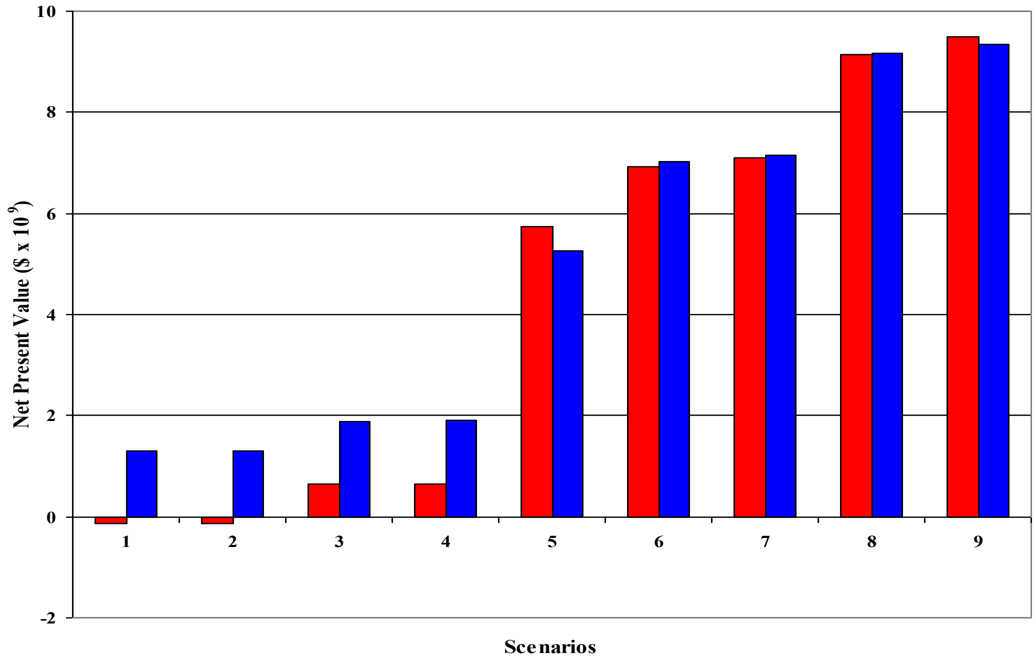
E[NPV] = $\$4.92 \times 10^9$



Solution proposes building 2 small FPSO's in the first year and then add new facilities / drill wells (recourse action) depending on the positive or negative outcomes.

Distribution of Net Present Value



Deterministic Mean Value = $4.38 x 10^9

Multistage Stoch Progr = $4.92 x 10^9 => ***12% higher and more robust***

Computation: Algorithm 1: 120 hrs; Algorithm 2: 5.2 hrs
Nonconvex MINLP: 1400 discrete vars, 970 cont vars, 8090 Constraints

Conclusions

1. **Effective solution of nonconvex MINLP and GDP requires tight lower bounds**

   Global optimization optimal water networks

2. **Energy and water optimization yields sustainable designs of biofuel plants**

   Optimization predicts lower energy and water targets

3. **Robustness can be effectively introduced with stochastic programming**

   Design of responsive supply chains, Multistage stochastic in oilfields