LARGE SCALE MIXED INTEGER GLOBAL OPTIMIZATION

Process Design and Systems Engineering Laboratory, Faculty of Technology, Åbo Akademi University
Laboratoriet för Anläggnings- och Systemteknik, Tekniska Fakulteten, Åbo Akademi

Process and Production Optimization Group, Professor Tapio Westerlund

Large scale

Large scale optimization problems are typically classified as problems containing several thousands of variables and constraints. These types of problems are becoming more and more common in industry as they are connected to great economical savings, and the methods and computers used to solve these problems are becoming more efficient.

In our case, we have focused on typical engineering optimization problems of large scale like process and production planning problems, as well as, logistics problems.

Mixed integer

Classical optimization theory deals only with problems modeled with continuous variables. However, in many real world industrial problems, such as process and production planning and logistics problems, discrete decisions need to be made. For example:
- Should machine A or machine B be used in a production plan?
- Which of three different types of heat exchangers should be used on a specific place in a flowsheet?
- How many reels of a specific width should be cut from a jumbo reel in a cutting pattern?

By using integer variables such decisions can be formulated in an exact way and the problems be modeled. The problem then becomes a mixed integer problem, including both continuous and integer variables. Problems including integer variables can, however, not be solved by classical optimization methods. Therefore new types of methods – mixed integer optimization methods – are being developed to solve these problems.

Global optimization

An optimization problem contains variables, an objective function to be minimized or maximized and constraints to be satisfied. As mentioned above, mixed integer problems include both continuous and integer variables. For the problem to be considered large, it typically contains thousands of variables as well as constraints.

The constraints, as well as, the objective function are in many cases linear, making the problem a mixed integer linear programming (MILP) problem. But in some cases, the problem at hand needs to be modeled by nonlinear expressions. Mixed integer optimization problems containing nonlinear expressions are called mixed integer nonlinear programming (MINLP) problems. The nonlinear expressions can further be classified in different classes. The most demanding class of optimization problems are nonconvex MINLP problems, since such problems may have several locally optimal solutions. To find the globally optimal solutions, so called global optimization method are created and used. These guarantee that a solution found is indeed the globally optimal one.

Nonconvex MINLP problems are quite common in engineering applications. For example, if an expression contains one or more bilinear terms (for example mass flow multiplied with temperature where both are unknown) the expression is already nonconvex.

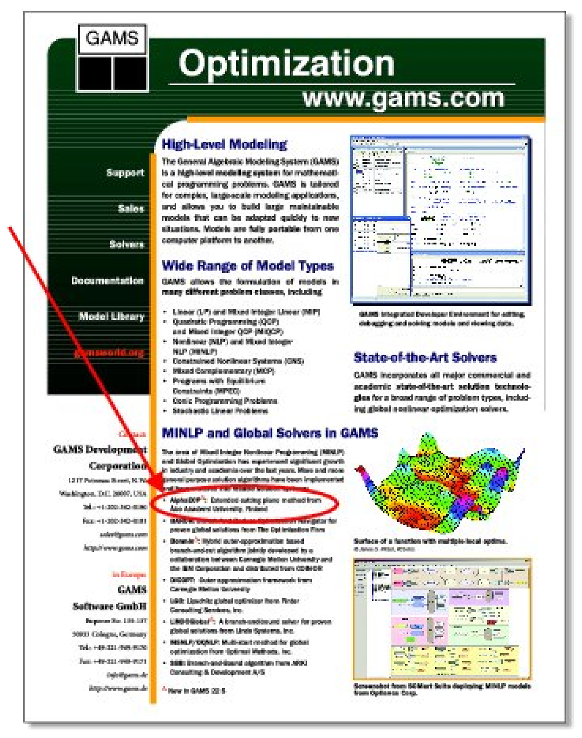
Some examples of the method and algorithm development, as well as, industrial applications done in the group are given below.

Method and algorithm development

The MINLP problem solver AlphaECP, developed in the group, is one of the state-of-the-art solvers available today on a worldwide basis for academic and industrial use, for example, via the GAMS modeling framework and the NEOS server for optimization.

The recently developed sequential cutting plane (SCP) algorithm is another MINLP solver, created in the group.

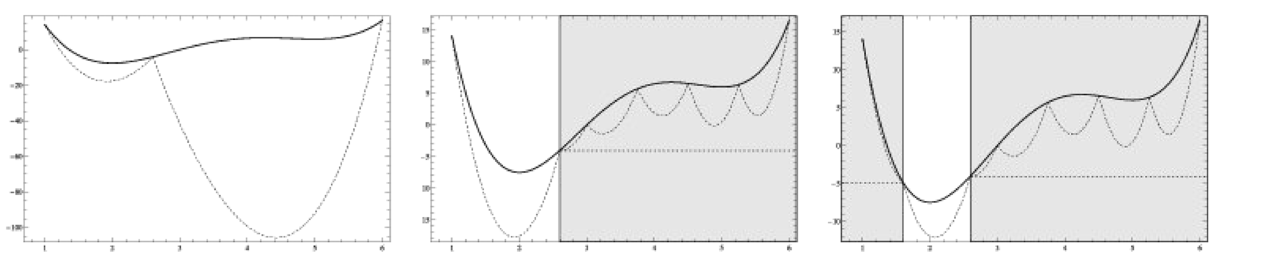
Methods for solving nonconvex MINLP problems containing so called signomial functions have been studied for some time, resulting in different advances in the area of convex underestimators for signomial terms. The global optimization algorithm, GGPECP, is an implementation of the results obtained.



Bussieck M., Lastusilta T. and Westerlund T. 2007

Global optimization methods are based on convexification and underestimation of nonconvex functions in a certain number of subregions. The transformed problem can for each subregion be solved to optimality with convex methods. By subsequently eliminating subregions and reducing the size of the remaining ones the global optimal solution of the nonconvex problem can be found.

In the methods developed in the group, the convexification of so called signomial functions is done by power or exponential transformations, and the underestimation by approximating the inverse transformations by piecewise linear functions.

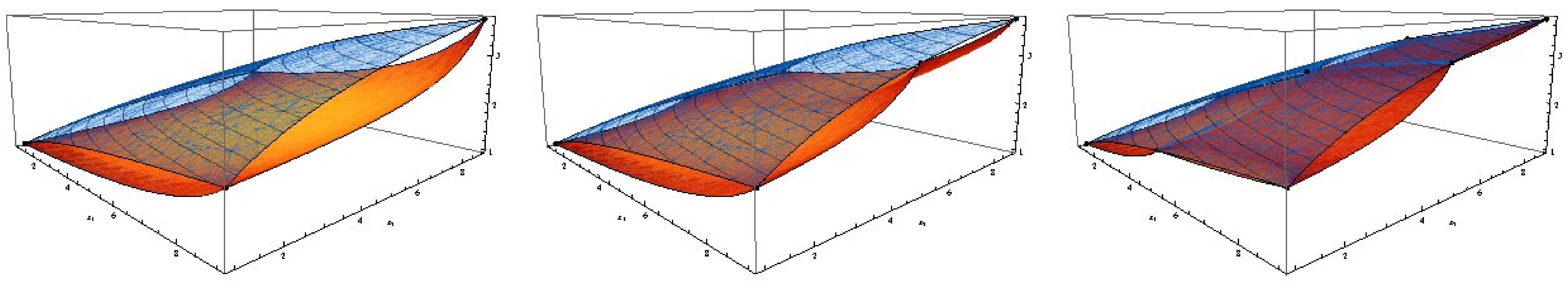


Underestimating a nonconvex function (the continuous line) in two subregions. The convexified underestimating function is indicated with the dashed line. From the figure we can see that none of the subregions can be eliminated at this stage. The best solution potentially in the subregion on the right-hand-side. Divide this subregion into new subregions.

The subregion on the right-hand-side in the previous figure has been divided into four additional subregions. From the figure, it can be observed that the function value at 2.6 is lower than any possible value in the shaded region, as this function value is below the smallest value of the underestimated convexified function. Thus, all subregions on the right-hand-side can be eliminated.

By dividing the subregion on the left-hand-side in the first figure into new subregions (two in the above figure), new subregions can be eliminated. By repeating the procedure of reducing and eliminating subregions the global optimal solution can finally be found.

Illustration of a nonconvex signomial term $x_1^{0.3} x_2^{0.3}$ convexified and underestimated in one, two and four subregions.



Lundell A., 2007

Academic results

Results from this research direction have been published in a large number of papers, mainly in journals like: *Optimization and Engineering, Journal of Global Optimization, Computers and Chemical Engineering, Engineering Optimization, European Journal of Operational Research, Global Optimization of Engineering Design, Applied Numerical Mathematics, Computational Optimization and Applications, Discrete Optimization, Computers and Operations Research.* Fourteen PhD students from the research group have finalized their theses since 1994.

Full list of papers available at *http://www.abo.fi/student/atartiklar*

Academic collaborators

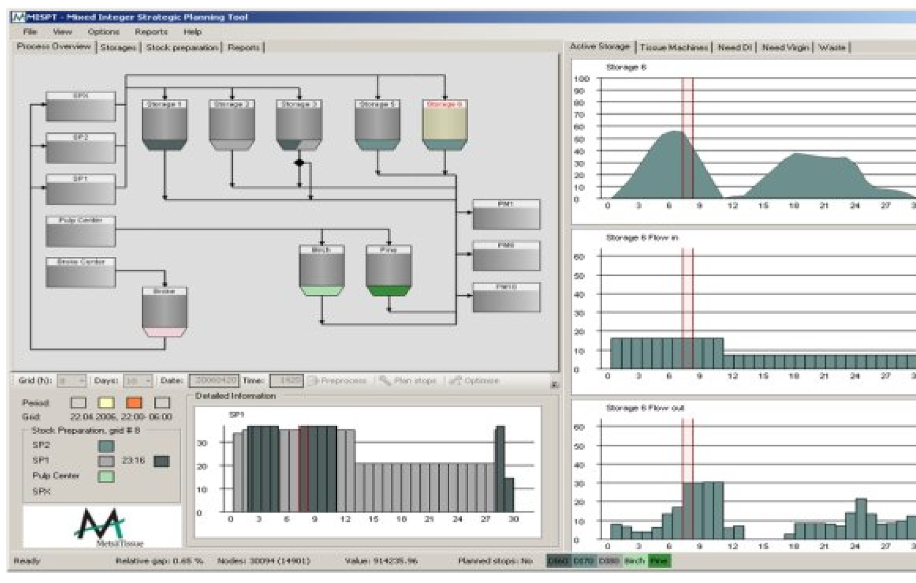
Beyond Nordic universities: *Princeton University (USA), Carnegie Mellon University (USA), University of Dundee (UK), Imperial College of Science and Medicine (UK), University College of London (UK), University of Braunschweig (GER), École Polytechnique Paris (FRA), Universitat Politecnica de Catalunya (ES), University of Maribor (SI), University of Alberta (CA)* and partners in the MINLP World forum.

Industrial applications

Several large scale optimization problems for which tailored solution systems and tools have been developed, in the group, are today in continuous industrial use.
- MISPT – A production planning tool for deinking lines. In continuous use at Metsä-Tissue Mänttä mill since 2005.
- MTS-2000 – A production optimization system for order handling and steel cutting. In continous use at Outokumpu Stainless, Tornio since 2000.
- WISA-Trim – Trim loss problem solver in the paper converting industry. In daily use at UPM-Walkiwisa plants in Pietarsaari and Steinfurt, since 1998 and Valkeakoski since 2004.
- ValmetOPT – Optimal design of heat recovery systems for paper machines, Metso 1999.
- Production scheduling in pharmaceutical industry, Schering.
- Optimization of supply chain for biofuels, Meriaura.
- Production planning, Danisco and Raniplast. (For more information see the SMARTOPT poster).

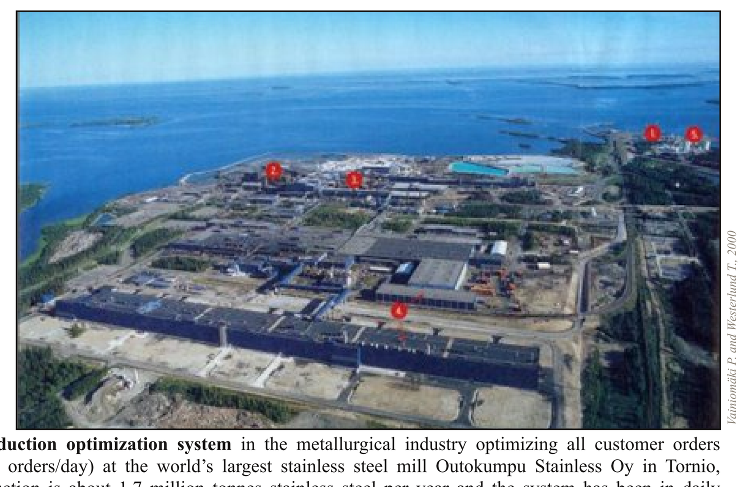
MISPT, an inventory planning and stock preparation system for scheduling the deinking and pulp lines for three tissue machines at Metsä-Tissue Mänttä Mill. The system has been in daily use since 2005.

Typical problem size is 12,500 variables and 15,000 constraints for a two week time horizon.



The MISPT interface

Outukumpu Stainless Tornio



Vainiomäki P. and Westerlund T., 2000

MTS-2000, a production optimization system in the metallurgical industry optimizing all customer orders (about 1,000-4,000 orders/day) at the world's largest stainless steel mill Outokumpu Stainless Oy in Tornio, Finland. The production is about 1.7 million tonnes stainless steel per year and the system has been in daily industrial use since 2000.

This application was a joint work with the engineering consulting company Accenture, which implemented our production planning models and solution methods in the VASU-system in Tornio as MTS-2000.

About 150,000,000 variables are optimized on a yearly basis, in this large scale optimization application using three 2.8 GHz PCs automatically solving about 400 optimization problems (with about 1,000 variables and constraints each) every day.

Acknowledgements

PhD Claus Still, PhD Per Jernström, PhD Joakim Westerlund, PhD Stefan Emet, PhD Kaj-Mikael Björk, PhD Janne Roslöf, PhD Jerker Björkqvist, PhD Stefan Karlsson, PhD Ray Pörn, PhD Nenad Milosavljevic, PhD Hans Skrifvars, PhD Iiro Harjunkoski, PhD Frank Pettersson, PhD Jarmo Söderman, MSc Päivi Holmi, MSc Toni Lastusilta, MSc Andreas Lundell, MSc Mattias Hästbacka, MSc Pekka Vainiomäki, MSc Johnny Isaksson, Mikael Nyberg, Oskar Frankenhauser.

Optimization of supply chain for biofuels

A MILP model with 775,000 variables and 730,000 constraints. Time horizon one year and time discretization one month. The model contains all the elements of the supply chain: factories, factory storages, inventory levels, transports from factory storages to ports and port storages, port fees, ship sizes, deplacements and ice classes, fuel consumption, reloadings, truck transports, available transport capacities, customers, monthly sale quantities, and so on.

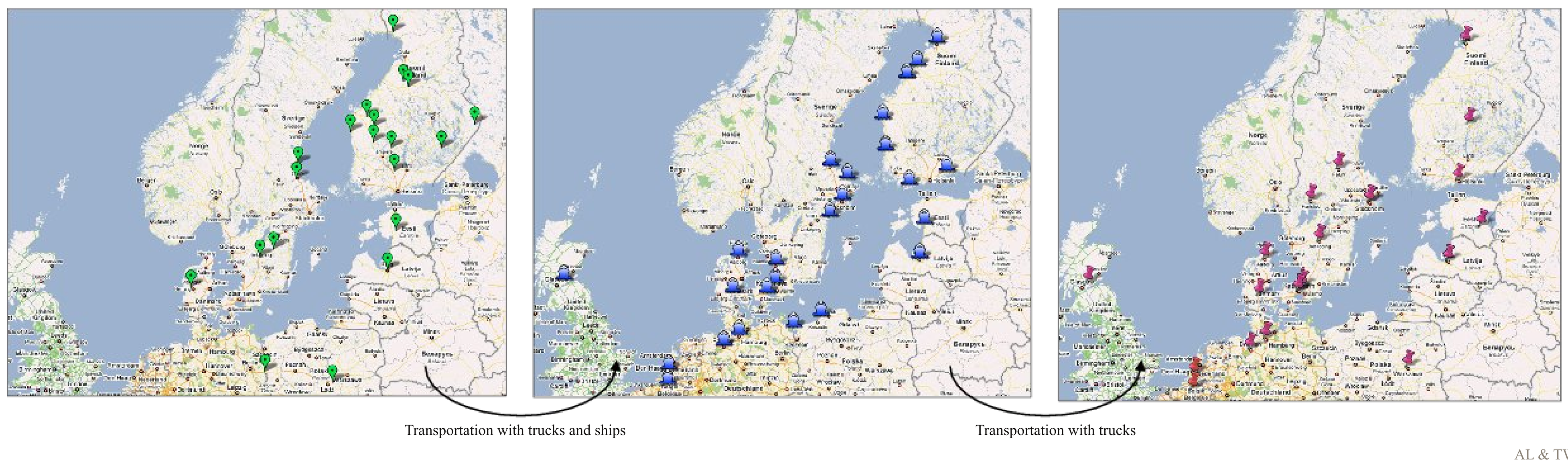
The biofuel transport to the customers originate from 19 factories (with different production capacities), each with a factory storage, in six different countries. The transports take place either by trucks or ships. The model contains 25 ports and 17 different port storages in eight countries. The land transports are with trucks of different sizes and the sea transports with cargo ships of different displacement classes. The customers consist mainly of so called consumer customers and power plant customers. The total annual sales of biofuels is in the current case about 808,000 tonnes.

The transports to the customers result, according to the solution, in about 39,000 trips by truck and a total of 3.3 million kilometers by land transport. About 36 % of the biofuel is transported directly from a factory storage to the customer. The rest is transported via additional storages or is transported directly via sea transports. A total of 504,000 tonnes of biofuel is transported by cargo ships, resulting in about 250 trips with a total transport length of 152,000 nautical miles. In the model, ten different ship sizes between 1,000 and 10,000 deadweight tonnage have been used. The solution also indicates the most favourable ship sizes.

19 factories and factory storages

25 harbors and 16 harbor storages

Customers



Transportation with trucks and ships

Transportation with trucks

AL & TW 2008