ABSTRACT

In this study, a number of symmetry-breaking (SB) constraints are proposed to improve the performance and decrease computational effort needed in the solution of process plant layout (PPL) problems. The PPL problems are formulated as Mixed-Integer Linear Programming (MILP) models in order to determine the efficiency of the SB formulations. Test runs are carried out on three different problem sets, both single-floor and multi-floor problems are used.

INTRODUCTION

Plant layout plays an important role in the design of industrial facilities. Patsiatzis and Papageorgiou (2003) the process plant layout (PPL) problem involves deciding concerning the spatial allocation of equipment items and the required space among them. The choosing mathematical models are combinatorial optimisation problems which usually require significant computational effort for their solution. It is evident that enrichment of the solution efficiency of PPL problems is a research topic of great relevance.

Symmetry in terms of multiple optimal solutions often consume additional CPU time in already tedious layout problems thus worsening the overall solution performance. Due to the geometry of the generalised PPL problem, symmetric layout solution alternatives will be selected for each problem implying the existence of multiple equivalent optimal solutions thus resulting in longer CPU time. Better efficiency in terms of CPU time can be obtained by breaking the symmetries, in this study a set of symmetry-breaking constraints are presented and empirically tested on different types of layout problems.

The pictures at the left are representing four equivalent symmetrical solutions for the same layout problem. The three equipment items, A, B and C, are allocated at different positions in each solution alternative but the solution value is the same for all symmetries. In these dimensional layout problems, the number of equivalent symmetrical solutions to the same problem is quite large. By using any of the SB constraints presented in this study, only one of the solutions could have been obtained.

PROBLEM FORMULATION

PPL problems can be formulated as non-convex MINLP or as discretised MILP problems according to Papageorgiou and Patsiatzis (2002). A number of rectangular units are to be sited in a rectangular plant area using a limited number of floors. The objective involves minimising either the joint connection cost or the total number of unit cost of each solution case. The detailed PPL problem is defined by resolving number of floors used, land area, floor allocation of each equipment item and detailed floor layout.

SYMMETRY BREAKING CONSTRAINTS

A number of different SB constraints are presented and empirically tested. Each constraint is tested on different kind of PPL problems and improvements of the solution performance is evaluated.

Symmetry-breaking constraint alternative 1.

The first SB constraint is designed to break the symmetry by pre-defining the relative allocation of two chosen equipment items in relation to each other. The relative allocation of the chosen equipment item i and item j is defined as:

\[ x_i - y_i - x_j - y_j \leq 0 \]

The sum of the \( x \) and \( y \) values of the centroid of unit \( i \) is defined to be less or equal to the corresponding sum for item \( j \), thus pre-defining the relative allocation of units \( i \) and \( j \). It is symmetric to the case of exclusion of the global optimal solution. SB constraint alternative 1 applied on Facility Layout problems is presented in Westerlund and Castillo (2002).

Symmetry-breaking constraint alternative 2.

The second SB constraint alternative 2 breaks the symmetry by forcing the centroid of a chosen equipment item \( i \) to be positioned in a specific corner of the whole facility area using the following constraints:

\[ x_i - \frac{1}{2} \leq x_{\text{tot}} \leq x_i + \frac{1}{2} \]

\[ y_i - \frac{1}{2} \leq y_{\text{tot}} \leq y_i + \frac{1}{2} \]

The solution value for problem \( n \). It is the global optimal solution and \( n \) the number of problems solved at CPU time. The CPU time of the lowest point on each profile shows the fastest solution time and the last point the solution time for the considered problem set thus making the ending of each profile the CPU time within which all problems in the considered set have been solved.

If a performance profile is on the left hand side of another, the first alternative solves the considered problem faster than the latter one. A performance profile ending up at a higher value than another indicates that the first alternative in average gives better solutions than the latter one.

ILLUSTRATIVE EXAMPLES

In this study, three different problem sets are solved to highlight the benefit of the considered SB constraints. The first set of illustrative problems consist of ten equipment items and five units each to be allocated on a single floor. In a plant are set-up of maximum three different floors. The second problem set is also termed as an urgent multi-floor problems but with a total of seven equipment items each. The last set of problems consist of basically the same problems as in the second set but the number of items used is limited to only one single-floor problem.

The first set of problems is solved using two different discretisation grids for the discrete linear constraints thus resulting in two alternative solutions for each problem. By solving problems using a fine discretisation grid, solution quality is prioritised on the expense of the solution time while problems solved using a coarse discretisation grid results in the opposite. To demonstrate how the SB constraints are affecting the solution time and quality of the considered problems, performance charts for each problem are sketched. The performance charts show the performance indicator over the CPU time. The CPU time is defined as follows:

\[ w_{\text{CPU_time}} = \frac{\sum w_i}{N} \]

where

\[ w_i = 1 - \frac{f_i}{f_{\text{opt}}} \]

\( f_i \) is the solution in question for problem \( n \). It is the global optimal solution and \( f_{\text{opt}} \) the number of problems solved at CPU time. The CPU time of the lowest point on each profile shows the fastest solution time and the last point the solution time for the considered problem set thus making the ending of each profile the CPU time within which all problems in the considered set have been solved.

If a performance profile is on the left hand side of another, the first alternative solves the considered problem faster than the latter one. A performance profile ending up at a higher value than another indicates that the first alternative in average gives better solutions than the latter one.

CONCLUSIONS

Symmetry breaking is an effective way of reducing computational efforts required to solve different kind of layout problems. The capacity of incorporating symmetry-breaking constraints with existing mathematical formulations for process plant layout has been illustrated through these different problems.

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