Improved Performance in Process Plant Layout Problems using Symmetry-Breaking constraints

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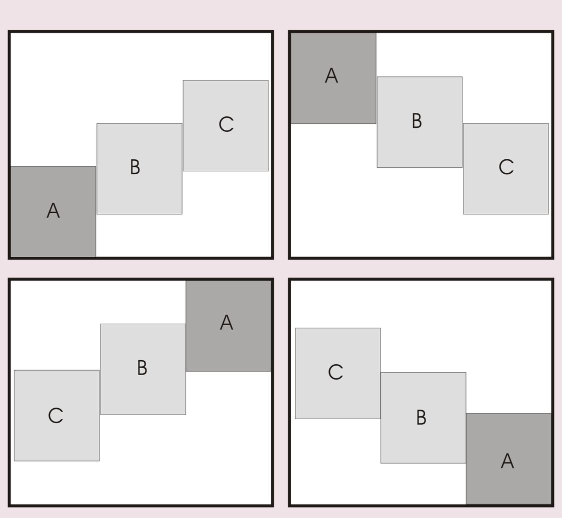
ABSTRACT

In this study, a number of symmetry-breaking (SB) constraints are considered to improve the performance and decrease computational effort needed in the solution of Process Plant Layout (PPL) problems. The PPL problems are formulated as Mixed-Integer Linear Programming (MILP)models. In order to determine the efficiency of the SB formulations, test runs are carried out on three different problem sets. Both single-floor and multi-floor problems are used.

INTRODUCTION

Plant layout plays an important role in engineering design of industrial facilities, Patsiatzis and Papageorgiou (2003). The process plant layout (PPL) problem involves decisions concerning the spatial allocation of equipment items and the required connections among them. The resulting mathematical models are combinatorial optimisation problems which usually require significant computational effort for their solution. It is evident that enhancement of the solution efficiency of PPL problems is a research topic of great relevance.

Symmetries in form of multiple optimal solutions often consume additional CPU time in already tedious layout problems thus worsening the overall solution performance. Due to the geometry of the generalised PPL problem, symmetric layout solution alternatives will be received for each problem implying the existence of multiple equivalent optimal solutions thus resulting in longer CPU time. Better efficiency in terms of CPU time can be obtained by breaking the symmetries. In this study a set of symmetry-breaking constraints are presented and empirically tested on different types of layout problems.



The pictures at the left are representing four equivalent symmetrical solutions for the same layout problem. The three equipment items, *A*, *B* and *C* are allocated at different positions in each solution alternative but the solution value is however the same for all alternatives. In three dimensional layout problems, the number of equivalent symmetrical solutions to the same problem is greater. By using any of the SB constraints presented in this study, only one of the solutions could have been obtained

PROBLEM FORMULATION

PPL problems can be formulated as non-convex MINLP or as discretised MILP problems according to Patsiatzis and Papageorgiou (2002). A number of rectangular units are to be sited in a rectangular plant area using a limited number of floors. The objective involves minimisation of layout costs and construction costs. The detailed PPL for a given problem is determined by resolving number of floors used, land area, floor allocation of each unit/equipment item and detailed floor layout.

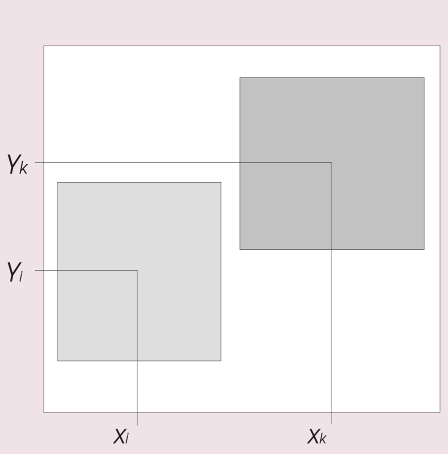
SYMMETRY BREAKING CONSTRAINTS

A number of different SB constraints are presented and empirically tested. Each constraint is tested on different kind of PPL problems and improvements of the solution performance is evaluated.

Symmetry-Breaking constraint alternative 1.

The first SB constraint alternative is designed to break the symmetry by pre-defining the relative allocation of two chosen equipment items in relation to each other. The relative allocation of the selected equipment items *i* and *k* is defined as,

$$x_i \quad y_i \quad x_k \quad y_k \quad 0$$



The sum of the *x*- and *y*-values of the centroid of unit *i* is defined to be less or equal to the corresponding sum for item *k*, thus pre-defining the relative allocation of units *i* and *k*. By pre-defining the relative allocation of items *i* and *k*, symmetric solutions are prevented without risk of exclusion of the global optimal solution. SB constraint alternative 1 applied on Facility Layout problems is presented in Westerlund and Castillo (2002).

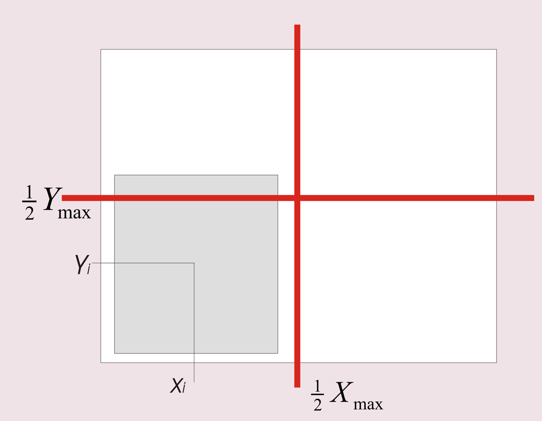
Symmetry-Breaking constraint alternative 2.

SB constraint alternative 2 breaks the symmetry by forcing the centroid of a chosen equipment item *i* to be positioned in a specific corner of the whole facility area using the following constraints,

$$x_i \quad \tfrac{1}{2} X_{\max} \quad 0 \qquad\qquad y_i \quad \tfrac{1}{2} Y_{\max} \quad 0$$

Nomenclature

x_i, y_i	Coordinates of geometrical centre of item *i*
$X_{\max}, Y_{\max}$	Dimensions of the facility floor area
l_i	Length of item *i*
d_i	Depth of item *i*
Z_{ik}	Binary variable, 1 if items *i* and *k* at same floor
$E1_{ik}$	Non-overlapping binary variable (as used in Papageorgiou and Rotstein, 1998)
	Parameter used in SB alt.4 to make constraints tighter
J_n^*	The global optimal solution for problem *n*
J_n	The solution value for problem *n*
w_n	Solution performance indicator for problem *n*
w_{tot}	Solution performance indicator for a set of problems



Item *i* is forced to be allocated in the lower left corner of the facility area thereby breaking the symmetry. SB constraint alternative 2 may not be used in combination with any of the other SB constraints as combining alternative 2 with alternatives defining the relative allocation of two equipment items may result in exclusion of the global optimal solution.

Symmetry-Breaking constraint alternative 3.

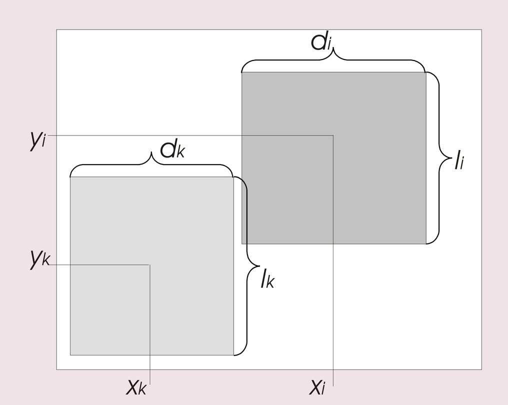
SB constraint alternative 3 is an further extension of alternative 1. In addition to the constraints used in alternative 1, alternative 3 also pre-defines one binary variable used in the Big-M constraints used to prevent unit overlap. Further information regarding the non-overlapping constraints used in the problem formulation is presented in Papageorgiou and Rotstein (1998) and Tsai et al. (1993).

The constraints used in SB alternative 3 are,

$$x_i \quad y_i \quad x_k \quad y_k \quad 0 \qquad\qquad E1_{ik} \quad 0$$

E1ik is one of two binary variables used in the non-overlapping constraints of each pair of units. The binary variables are used in so called big-M constraints used to formulate the disjunctive non-overlapping constraints in a linear form. The particular binary variable used in the considered constraint will activate one of two constraints displayed below when *E1ik* is defined equal to zero,

$$y_i \quad y_k \quad \frac{d_i \quad d_k}{2} \qquad\qquad x_i \quad x_k \quad \frac{l_i \quad l_k}{2}$$



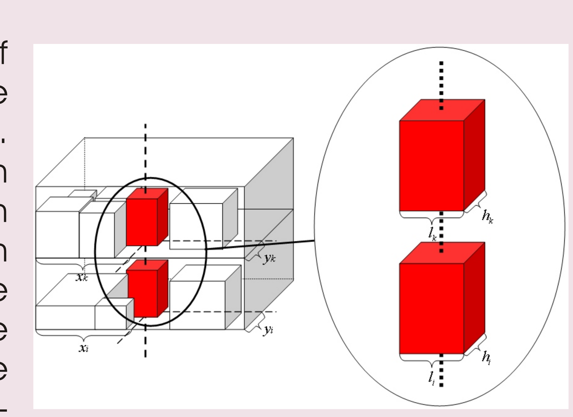
SB constraint alternative 3 locks unit *i* in a compass point NE (North-East) of unit *k* as illustrated in the figure. Unit *i* may also be locked in position NW, SE or SW of unit *k* by corresponding constraints. The symmetry-breaking considerations used in SB alternative 3 are based on the symmetry-breaking formulations applied to Facility Layout problems presented in Castillo and Westerlund (2002).

Symmetry-Breaking constraint alternative 4.

SB constraint alternative 4 is a tighter version of SB constraint alternative 3. To make the constraints tighter, a variable is included in the constraints. To prevent exclusion of the global optimal solution in multi floor process plant layoutproblems, a variable *Zik* is also included. The constraints used are,

$$x_i \quad y_i \quad x_k \quad y_k \qquad Z_{ik} \qquad\qquad E1_{ik} \quad 0$$

Zik is a binary variable with the value of 1 if equipment items i and k are allocated on the same floor, 0 otherwise. If variable *Zik* would not be included in the constraint, the global optimal solution could be excluded by the constraint in the specific case showed in the picture beside. The picture above illustrates the specific case where items *i* and *k* are allocated to the same x- and y-coordinates but at different floors at global optimum. Parameter is defined as the sum of half the length, *l* or the depth, *d* (the ones that are smaller) of both items used in the constraint according as shown below,



$$\min(\tfrac{1}{2} l_i, \tfrac{1}{2} d_i) \quad \min(\tfrac{1}{2} l_k, \tfrac{1}{2} d_k)$$

Three options of SB constraint alternative 4 are investigated and presented. The difference between the three alternatives is how the two items *i* and *k* used in the constraints are chosen. The first alternative, 4a, uses the two items with the largest joint connection cost. The second alternative, 4b, utilises the two smallest units while the third alternative, 4c, uses the two largest units.

A similar SB strategy as alternative 4, applied on facility layout problems is presented in Sherali et al. (2002).

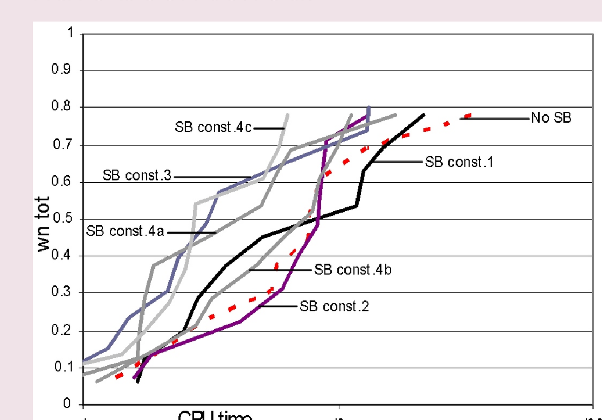
ILLUSTRATIVE EXAMPLES

In this study, three different problem sets are solved to highlight the benefit of the considered SB constraints. The first set of illustrative problems consist of ten slightly divergent multi-floor PPL problems with five units each to be allocated in a plant are set-up of maximum three different floors. The second problem set is also formed by ten divergent multi-floor problems but with a total of seven equipment items each. The last set of problems consist of basically the same problems as in the second set but the number of floors used is limited to only one (single-floor problems).

The first set of problems is solved using two different discretisation grids for the discretised linear area constraints thus resulting in two alternative solutions for each problem. By solving problems using a fine discretisation grid, solution quality is prioritised on the expense of the solution time while problems solved using a coarse discretisation grid results in the opposite.To demonstrate how the SB constraints are affecting the solution time and quality of the considered problems, performance charts for each problem are sketched. The performance charts show the performance indicator w_{tot} versus the CPU time. w_{tot} (displayed on the vertical axis in the charts) is defined as follows,

$$w_{tot} \quad \sum_{n\;1}^{N} w_n \; \frac{1}{N} \qquad \text{Where} \qquad w_n \quad 1 \quad \frac{J_n \quad J_n^*}{J_n^*}$$

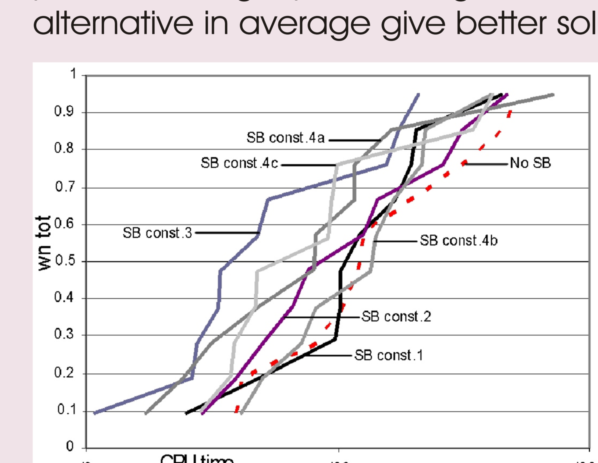
J_n is the solution in question for problem *n*, J_n^* is the global optimal solution and *N* the number of problems solved at CPU time *t*. The CPU time of the lowest point on each profile shows the fastest solution time and the last point the slowest solution time for the considered problem set thus making the endpoint of each profile the CPU time within which all problems in the considered set have been solved.



Solution data for the first set of PPL problems (five equipment items to be allocated at maximum three floors. (solved using a coarse grid).

	CPU time							Solution	Global
Ex	No SB	Alt.1	Alt.2	Alt.3	Alt.4a	Alt.4b	Alt.4c	Quality[%]	Optimum
1_1	5.5	2.8	6.0	3.4	5.7	11.3	5.9	113	72649
1_2	7.6	15.0	13.2	13.2	6.6	8.0	5.0	135	94452
1_3	3.0	3.7	7.0	2.4	1.7	8.4	2.8	111	174654
1_4	33.7	21.7	8.3	13.1	5.0	10.0	6.3	114	415672
1_5	8.6	5.1	9.0	6.4	3.1	6.2	2.2	121	77740
1_6	13.0	11.8	8.8	1.5	1.9	1.9	2.5	114	85462
1_7	1.9	1.8	1.9	0.8	0.8	1.1	1.4	138	31274
1_8	7.9	12.7	4.1	3.1	16.8	4.8	2.7	109	387644
1_9	1.4	2.5	1.6	2.2	1.7	3.2	0.6	128	53800
1_10	5.8	1.6	8.4	1.3	1.6	2.8	1.8	136	53532
Median	*6.7*	*4.4*	*7.6*	*2.7*	*2.5*	*5.5*	*2.6*		

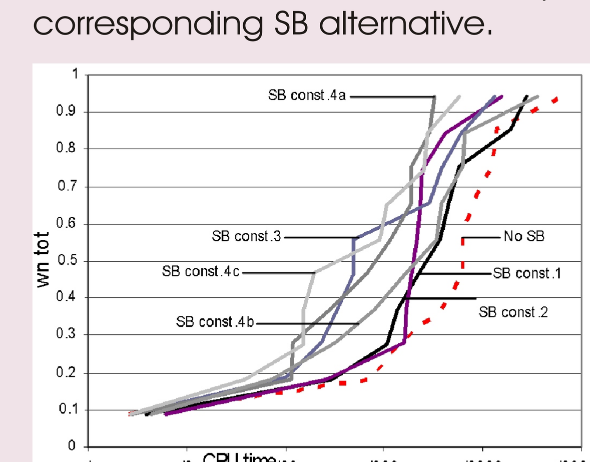
If a performance profile is on the left hand side of another, the first alternative solves the considered problem faster than the latter one. A performance profile ending up at a higher value than another indicates that the first alternative in average give better solutions than the latter one.



Solution data for the first set of PPL problems (five equipment items to be allocated at maximum three floors. (solved using a fine grid).

	CPU time							Solution	Global
Ex	No SB	Alt.1	Alt.2	Alt.3	Alt.4a	Alt.4b	Alt.4c	Quality[%]	Optimum
1_1	120.4	98.0	76.4	34.3	80.8	135.0	99.3	101	72649
1_2	314.1	445.5	261.9	156.9	116.0	406.2	91.1	108	94452
1_3	126.1	122.0	51.2	53.1	81.5	166.4	28.6	103	174654
1_4	195.9	194.6	126.6	173.6	164.9	214.6	342.1	105	415672
1_5	103.6	102.5	66.0	10.9	116.2	72.2	39.2	107	77740
1_6	82.9	48.6	39.8	26.3	24.5	82.3	48.0	106	85462
1_7	41.5	25.2	29.1	27.8	17.2	50.3	47.3	105	31274
1_8	498.8	168.6	311.0	47.9	48.8	141.2	95.1	104	387644
1_9	458.8	204.9	465.3	210.0	716.5	223.4	413.6	108	53800
1_10	39.4	101.8	144.9	33.3	31.5	41.7	37.6	107	53532
Median	*123.2*	*112.2*	*101.5*	*41.1*	*81.1*	*138.1*	*69.5*		

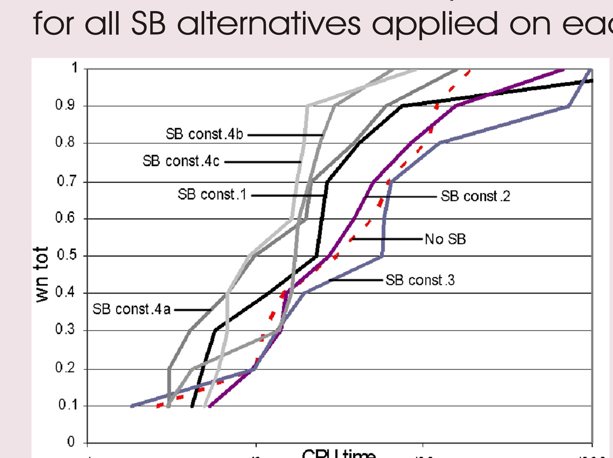
The endpoint of the performance profile in vertical direction gives the fractional average reached of the global optimum for all problems in the problem set concerned. The endpoint in time/horisontal direction indicates the CPU-time within which all problems in the set were sold using the corresponding SB alternative.



Solution data for the second set of PPL problems (seven equipment items to be allocated at maximum three floors).

	CPU time							Solution	Global
Ex.	No SB	Alt.1	Alt.2	Alt.3	Alt.4a	Alt.4b	Alt.4c	Quality[%]	Optimum
2_1	766.4	294.6	257.7	482.1	113.9	74.6	39.0	109	46742
2_2	3834.4	1083.8	1881.5	497.6	294.8	1628.2	153.7	106	68383
2_3	64875.2	27286.7	1665.6	336.4	1172.4	816.4	911.2	109	43330
2_4	1472.7	1357.2	2129.5	232.0	689.6	3439.4	156.7	107	63425
2_5	14074.5	2119.4	2366.0	3763.9	2895.2	6459.4	1054.3	108	67047
2_6	12517.1	19549.5	15366.8	6104.7	1919.8	35257.3	2515.5	103	49569
2_7	6379.7	4492.4	4089.0	2918.1	1932.9	6217.8	5841.6	101	93604
2_8	6311.0	3716.4	1642.2	100.8	119.1	328.1	198.0	102	54876
2_9	2.6	4.0	6.1	2.8	2.8	4.5	2.7	111	13068
2_10	8110.5	5724.8	2402.2	13396.2	3259.2	3803.6	2716.1	102	123585
Median	*6345.35*	*2917.87*	*2005.48*	*489.885*	*931*	*2533.8*	*554.6*		

The global optimum of each problem is obtained with the global optimisation code GGPECP, Westerlund and Westerlund (2003) using the non-convex MINLP formulation presented in Patsiatzis and Papageorgiou (2002). The global optimum of each problem is shown in the last column of the tables. Since the objective is to minimise the total plant cost for each problem, solution quality values above 100% indicates that the global optimum is not reached. The solution quality values shown in the tables are not affected by the SB constraints and are therefore the same for all SB alternatives applied on each problem.



Solution data for the single-floor PPL problems (seven equipment items to be allocated at one floor).

	CPU time							Solution	Global
Ex.	No SB	Alt.1	Alt.2	Alt.3	Alt.4a	Alt.4b	Alt.4c	Quality[%]	Optimum
SF_1	11.4	5.8	5.3	733.0	3.0	4.2	6.8	100	9948.0
SF_2	31.1	22.8	27.5	9.9	20.1	21.1	6.9	100	11740
SF_3	119.8	73.4	50.1	13.6	4.1	17.9	20.5	100	5264
SF_4	9.9	4.8	15.3	64.7	6.8	13.7	6.0	100	13465
SF_5	63.2	26.6	84.5	56.2	38.1	29.1	16.5	100	11774
SF_6	194.0	41.4	38.3	966.7	59.6	64.9	17.7	100	8716
SF_7	50.0	3192.3	682.9	19.3	21.5	24.1	89.3	100	16769
SF_8	14.9	11.9	14.1	124.1	156.3	16.4	19.3	100	10049
SF_9	2.6	4.2	9.6	1.8	3.1	3.0	5.0	100	2074
SF_10	105.2	24.7	154.6	58.3	9.9	17.1	9.2	100	23953
Median	*40.5*	*23.8*	*32.9*	*57.2*	*15.0*	*17.5*	*12.8*		

CONCLUSIONS

Symmetry-breaking is an effective way of reducing computational efforts required to solve different kind of layout problems. The benefit of incorporating symmetry-breaking constraints within existing mathematical formulations for process plant layout has been illustrated through three different problem sets.

ACKNOWLEDGEMENTS

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