A Comparison of Three Different Modeling Approaches for Solving Multi-Product, Multi-Purpose Plant Scheduling

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Many new production facilities are designed with no intermediate storage. Automated production equipment used usually presents a blocking behaviour. In many cases it also takes some time to empty a machine before the next task may start. This can be modeled as a blocking flowshop with move-out times. All formulations used in this comparison are of MILP type. The objective function has been the shortest completion time in all cases.

Discrete time model using a State-Task Network

In the State-Task Network a product in one state is transformed into another state by a task. All tasks must coincide with the discrete moments of time set by the time grid. The duration and move-out times both have their corresponding constraints.

\[
\sum_{i=1}^{n} W_{ij} + \sum_{t=1}^{T} W_{j,t} \leq 1, \forall i, j, t
\]

For each equipment there is also a storage term. For a blocking flowshop the storage has a maximum of one unit.

\[
S_{i} = S_{i,0} + W_{i,0} - W_{i,T}
\]

Time slot model using a Resource-Task Network

The Resource-Task Network is more flexible than the STN. In the RTN everything, like raw material and equipment, is handled as resources. In the STN material changes state in each task, but in the RTN it may also be equipment that changes state.

The use of a continuous time grid makes the model more flexible. The downside of this model implementation is that it is more complicated than the other two models studied.

Continuous time model using a disjunctive formulation

This model is strictly time based. Each task is either before or after another task. This can be written in disjunctive form for each pair of tasks.

\[
T_{ij} \geq T_{i,j+1} + \theta_{t} \quad \forall i, j
\]

Disjunctions can be rewritten in MILP format. The model has two parts. One for describing allocations on each equipment, and the other one describing the internal precedence and timing constraints.

\[
T_{j} + M \cdot (1 - y_{j}) \geq T_{i,j+1} + \theta_{t} \\
T_{j} + M \cdot (1 - y_{j}) \geq T_{i,j+1} + \theta_{t}
\]

Conclusions

All models have their strengths and weeknesses. The disjunctive model is usually the fastest of these three, but it cannot easily handle inventories like the two other models. The choice of objective function also affects performance to a great extent.

Nomenclature

- $W_{ij}$: Discrete variable noting the start of task $i$ on equipment $j$ at time $t$. $1$ if task starts, otherwise $0$
- $S_{i}$: Discrete variable for equipment allocation at time $t$. $1$ if allocated, $0$ if not.
- $t$: Time it takes to complete one production step
- $T_{ij}$: Continuous variable noting the start of task $i$ on equipment $j$. $M$: Maximum makespan
- $y_{j}$: Binary variable, $1$ if statement true, $0$ if false.
- $i$: Index referring to a task
- $j$: Index referring to equipment
- $T$: Index referring to time slot
- $a,b$: Index referring to different products

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