Monte Carlo Methods for Early-exercise Options

Christian Bender (Saarland University)

Abstract

In this lecture series we discuss numerical methods for the pricing of Bermudan options via Monte Carlo simulation. A Bermudan option gives the holder the right to exercise a cash-flow once. The holder may choose the exercise time from a finite set of time points $\mathcal{E}$, i.e. she has to choose a $\mathcal{E}$-valued stopping time as her exercise time. The pricing problem of these Bermudan options can be formulated as an optimal stopping problem (in discrete time, although the option may be written on underlying assets that evolve in continuous time). The formulation as an optimal stopping problem gives rise to a dynamic program. The numerical solution of this dynamic program is a hard problem, particularly when the option is written on a high-dimensional underlying. In such situation simulation methods are required. However, while simulation is naturally directed forward in time, the dynamic program is directed backward and requires a nested evaluation of conditional expectations.

In recent years several simulation methods have been developed in order to obtain estimators for the option price which a biased low or high, respectively. They can be combined to calculate confidence intervals for the Bermudan option price. An approximate backward dynamic program based on least-squares Monte Carlo estimators for the conditional expectations can be employed to construct a 'close-to-optimal' exercise strategy, which gives rise to a lower bound for the option price (Longstaff-Schwartz algorithm). Via duality theory (Rogers, Haugh/Kogan) upper bounds can be calculated from the lower bound. In this respect we present the Andersen-Broadie algorithm, which requires nested simulation, and an upper bound algorithm based on martingale representation properties, which avoids nested simulations (Belomestny/B./Schoenmakers). The algorithms are illustrated by numerical examples. Finally we discuss extensions to option which can be exercised several times (e.g. swing options in electricity markets).

Andersen, L., M. Broadie (2004): A Primal-dual Simulation Algorithm for Pricing Multidimensional American Options. *Management Sciences*, 50, 1222–1234.

Belomestny, D., C. Bender, and J.G.M. Schoenmakers (2009): True Upper Bounds for Bermudan Products via Non-Nested Monte Carlo. *Mathematical Finance*, 19, 53–71.

Haugh, M., L. Kogan (2004): Pricing American Options: a Duality Approach. *Operations Research*, 52, 258–270.

Longstaff, F.A., E.S. Schwartz (2001): Valuing American Options by Simulation: A Simple Least-squares Approach. *Review of Financial Studies*, 14, 113–147.

Rogers, L.C.G. (2001): Monte Carlo Valuation of American Options. *Mathematical Finance*, 12, 271–286.